Международная конференция
Физика атмосферы и климата
Посвящённая 70-летию Института физики атмосферы им. А.М.Обухова
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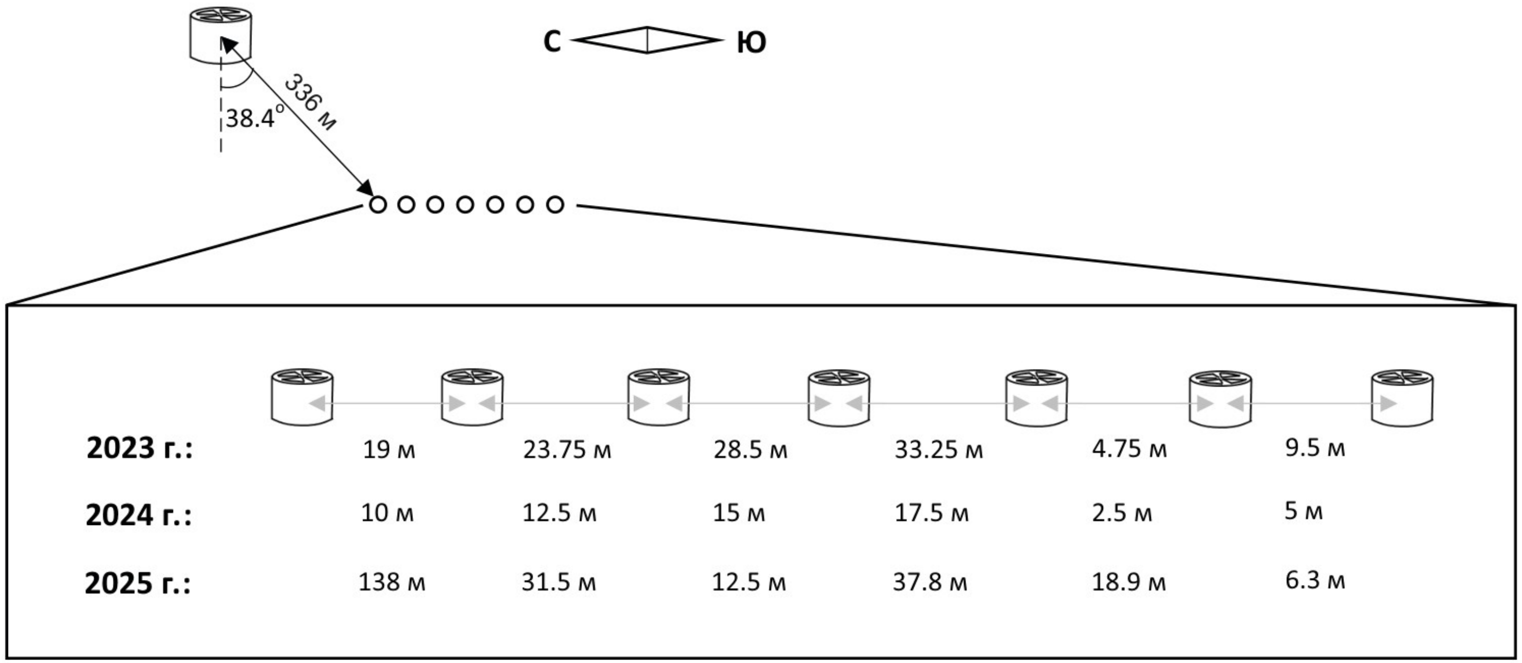
# Корреляционно-спектральные модели турбулентных флуктуаций атмосферного электрического роля по результатам натурных наблюдений и вихреразрешающего моделирования

# Correlation-spectral models of turbulent fluctuations of the atmospheric electric field based on the results of field observations and large-eddy simulation

С.В. Анисимов, С.В. Галиченко, А.А. Прохорчук, Е.В. Климанова

Институт физики Земли им. О.Ю. Шмидта РАН
Геофизическая обсерватория «Борок»

# Field observations




119 м

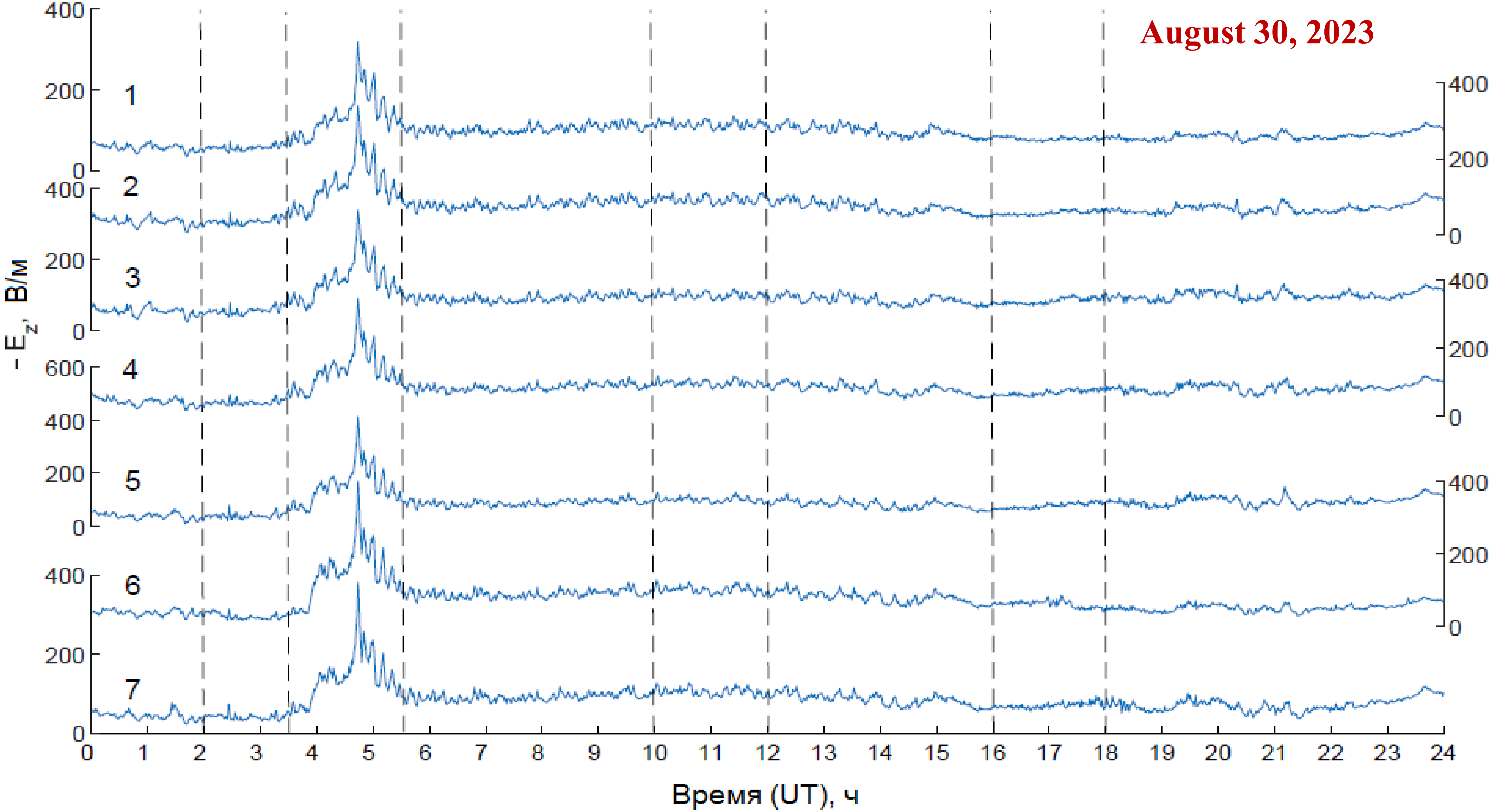
62.5 м

245 м

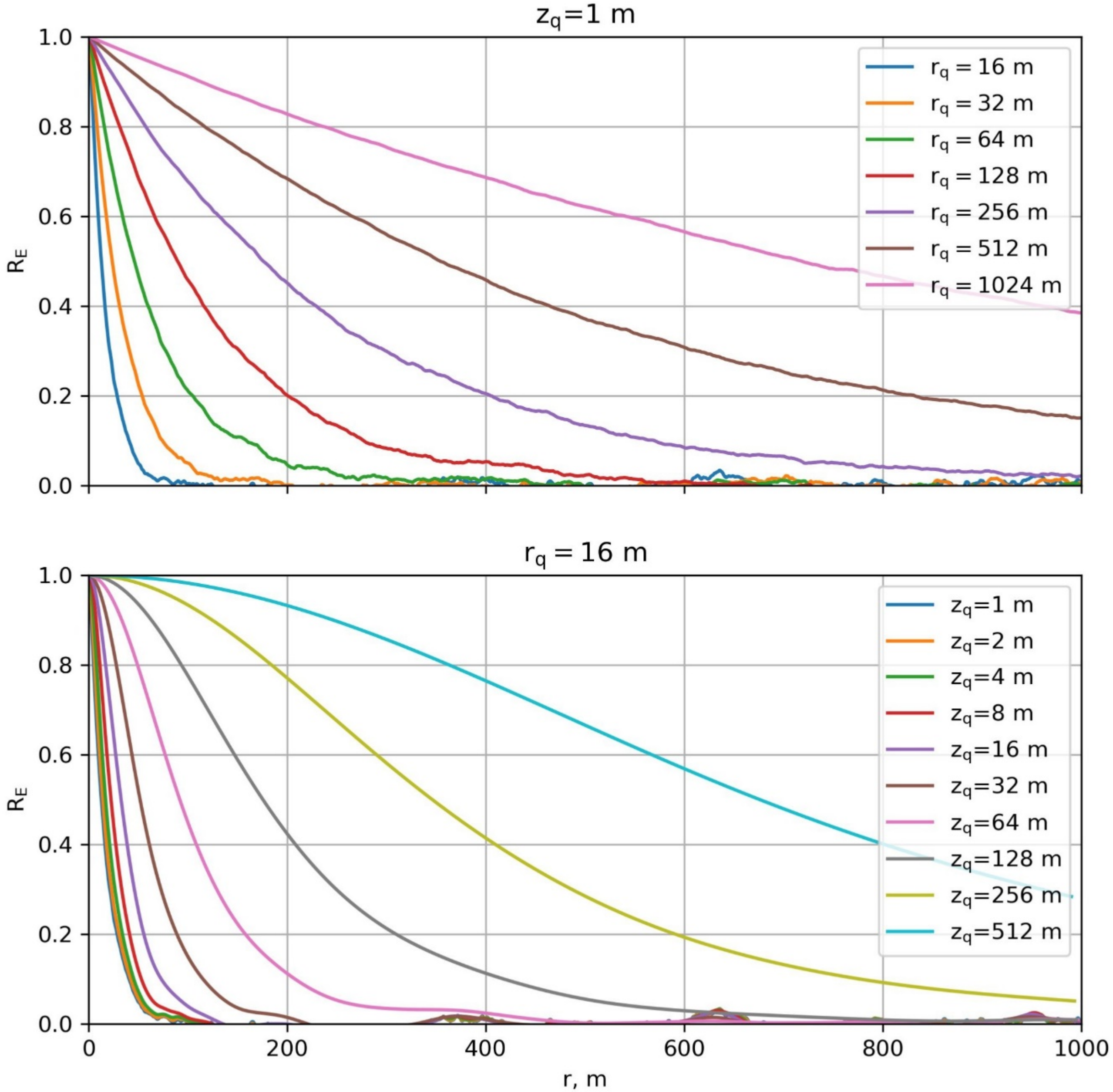
**Линейки Голомба 7 порядка**
**Golomb rulers of 7th order**

0-2-3-10-16-21-25
0-1-4-10-12-17-39

# Synchronous multipoint recordings $E_z$



# One-dimensional q-model

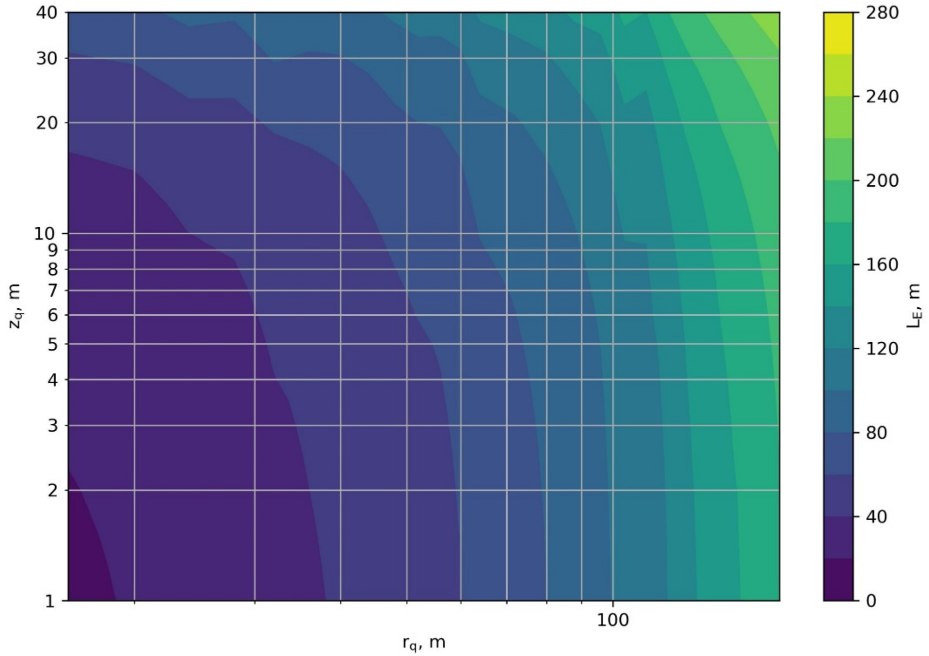



$$R_E\left(r_{ij}\right)=\frac{\left\langle\left(E_z\left(x_i\right)-\left\langle E_z\left(x_i\right)\right\rangle\right)\left(E_z\left(x_j\right)-\left\langle E_z\left(x_j\right)\right\rangle\right)\right\rangle}{\sigma_E\left(x_i\right)\sigma_E\left(x_j\right)}, \quad r_{ij}=\left|x_i-x_j\right|$$

**Families of ACF $E_z$ calculated in the model with a line with Gaussian randomly distributed charges with exponential ACF at a height $z_q$ above the conducting plane**

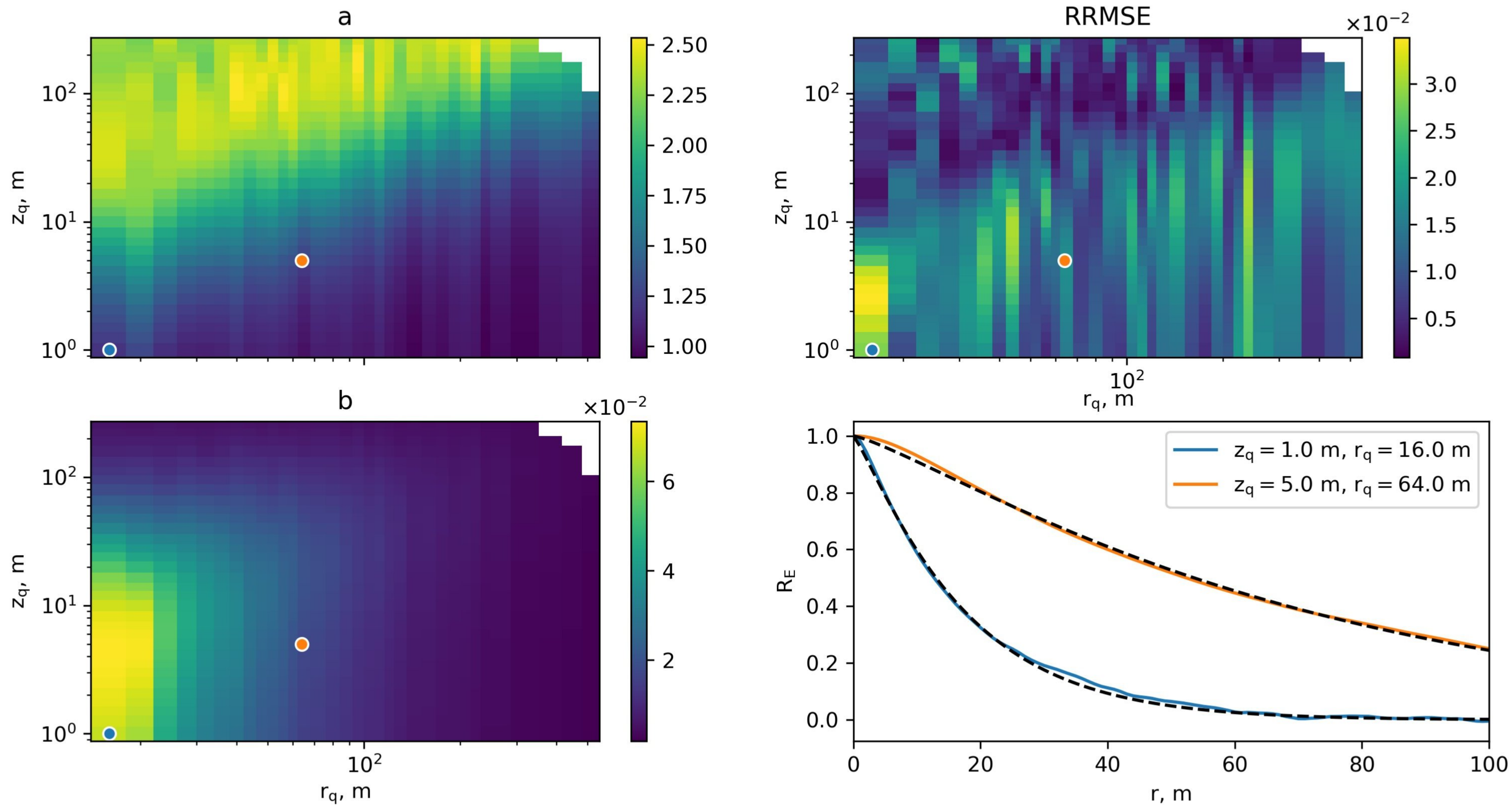
**$r_q$ is the correlation radius in the line of charges**

**Integral scale** $L_E\left(r_c\right)=\int_0^{r_c} R_E(r)\,dr, \quad R_E\left(r_c\right)=R_c>0$

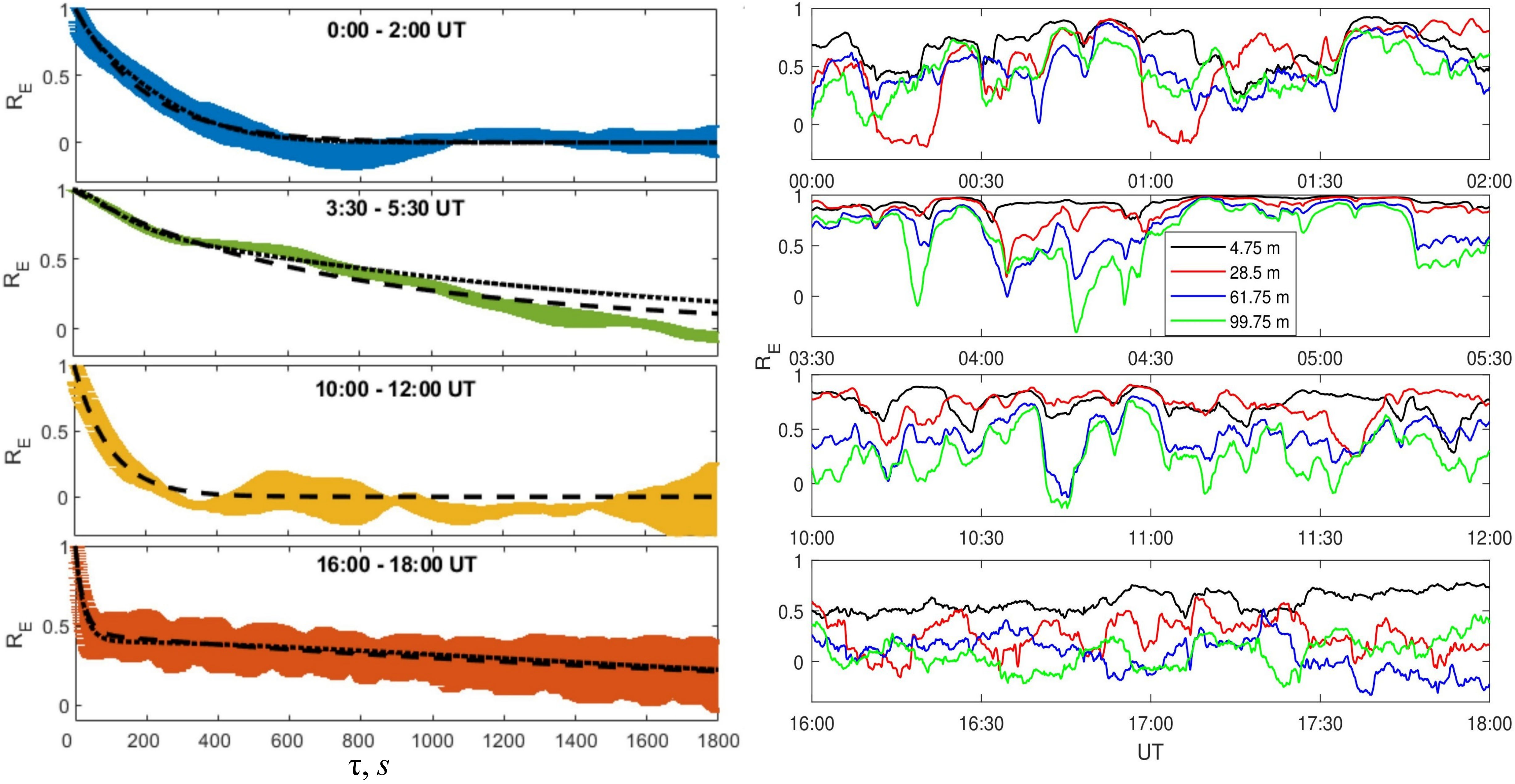


# Gamma distribution tail model for ACF $R_E(r) = \frac{\Gamma(a, br)}{\Gamma(a)}$

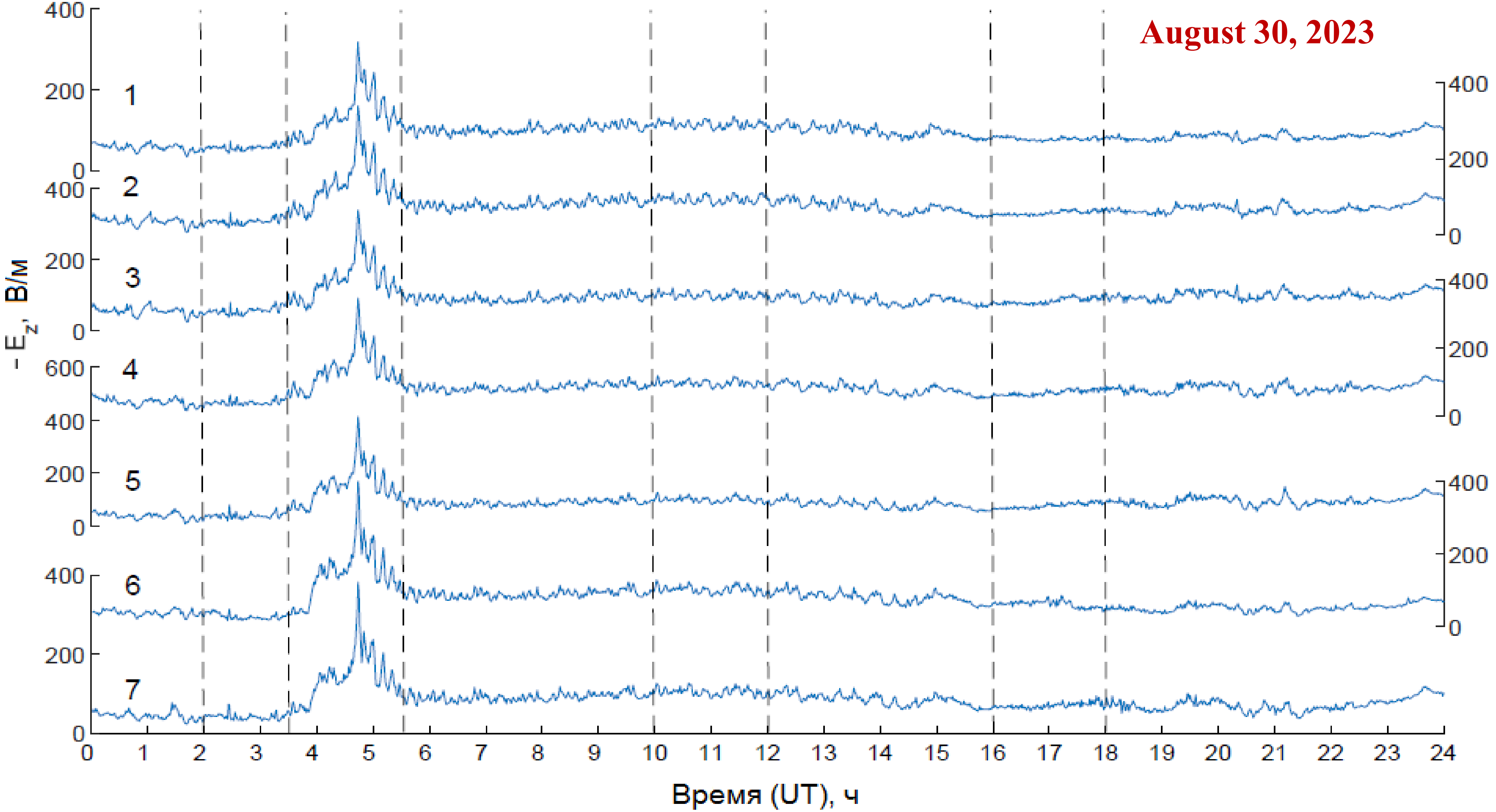
$$X \sim \Gamma(k,\theta), k>0, \theta>0, \quad f(x) = \begin{cases} \frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\theta^k}, & x \ge 0 \\ 0, & x \le 0 \end{cases}, \quad \overline{F}(x) = P(X>x) = \frac{\Gamma(k, x/\theta)}{\Gamma(k)}, \quad \Gamma(k, x/\theta) = \int_{x/\theta}^{\infty} t^{k-1}e^{-t}dt$$



# Temporal and spatial correlations of $E_z$ fluctuations



# Synchronous multipoint recordings $E_z$



# Models of time ACF (*R*) and frequency PSD (*S*)

$$R(\tau)=\int_0^\infty S(\omega)\cos\omega\tau d\omega,\qquad S(\omega)=\frac{2}{\pi}\int_0^\infty R(\tau)\cos\omega\tau d\tau$$

1) $R_1(\tau;a)=\dfrac{\Gamma(a,b|\tau|)}{\Gamma(a)}$ $\qquad S_1(\omega;a)=\dfrac{2}{\pi}\displaystyle\int_0^\infty R_1(\tau;a)\cos\omega\tau d\tau=\dfrac{2\sin\left[a\cdot\operatorname{arctg}(\omega/b)\right]}{\pi\omega\left[1+(\omega/b)^2\right]^{a/2}},\quad 0<a\le 2$

2) $R_2(\tau;\nu)=2\left(\dfrac{b|\tau|}{2}\right)^{\nu}\dfrac{K_\nu(b|\tau|)}{\Gamma(\nu)}$ $\qquad S_2(\omega;\nu)=\dfrac{2b^{2\nu}\Gamma(\nu+1/2)}{\sqrt{\pi}\Gamma(\nu)(b^2+\omega^2)^{\nu+1/2}},\quad \nu,b>0$

3) $R_3(\tau;\beta)=1-c|\tau|^{\beta}e^{-b|\tau|}$ $\qquad S_3(\omega;\beta)=\pi\delta(\omega)-\dfrac{c\Gamma(\beta+1)\cos\left[(\beta+1)\cdot\operatorname{arctg}(\omega/b)\right]}{(b^2+\omega^2)^{\frac{\beta+1}{2}}},\quad c,\beta,b>0$

# Relationships between ACF-PSD models 1 and 2

## Integral scale

$$T_1 = \int_0^\infty R_1(\tau; a)\, d\tau = \frac{a}{b} \qquad T_2 = \int_0^\infty R_2(\tau; \nu)\, d\tau = \frac{\Gamma(1/2)\Gamma(\nu + 1/2)}{b\Gamma(\nu)}$$

## Matching smoothness parameters

$$a = \frac{\pi}{\mathrm{B}(1/2, \nu)} = \frac{\sqrt{\pi}\Gamma(\nu + 1/2)}{\Gamma(\nu)}$$

$$\nu \approx \left(\frac{\ln \zeta/3 + a}{\sqrt{\pi}}\right)^{\zeta}, \qquad \zeta = 1.85$$

## Special parameters values

$$R_2\left(x; \frac{1}{2}\right) = \frac{2K_{1/2}(x)}{\Gamma(1/2)}\sqrt{\frac{x}{2}} = e^{-x} = R_1(x; 1),$$

$$R_2\left(x; \frac{3}{2}\right) = \frac{2K_{3/2}(x)}{\Gamma(3/2)}\left(\frac{x}{2}\right)^{3/2} = e^{-x}(1 + x) = R_1(x; 2).$$

## Inflection points

$$b\tau_2 = (2\nu - 1)\frac{K_{\nu-1}(b\tau_2)}{K_\nu(b\tau_2)}, \quad \frac{b\tau_2}{a - 1} - 1 \approx 0.06$$

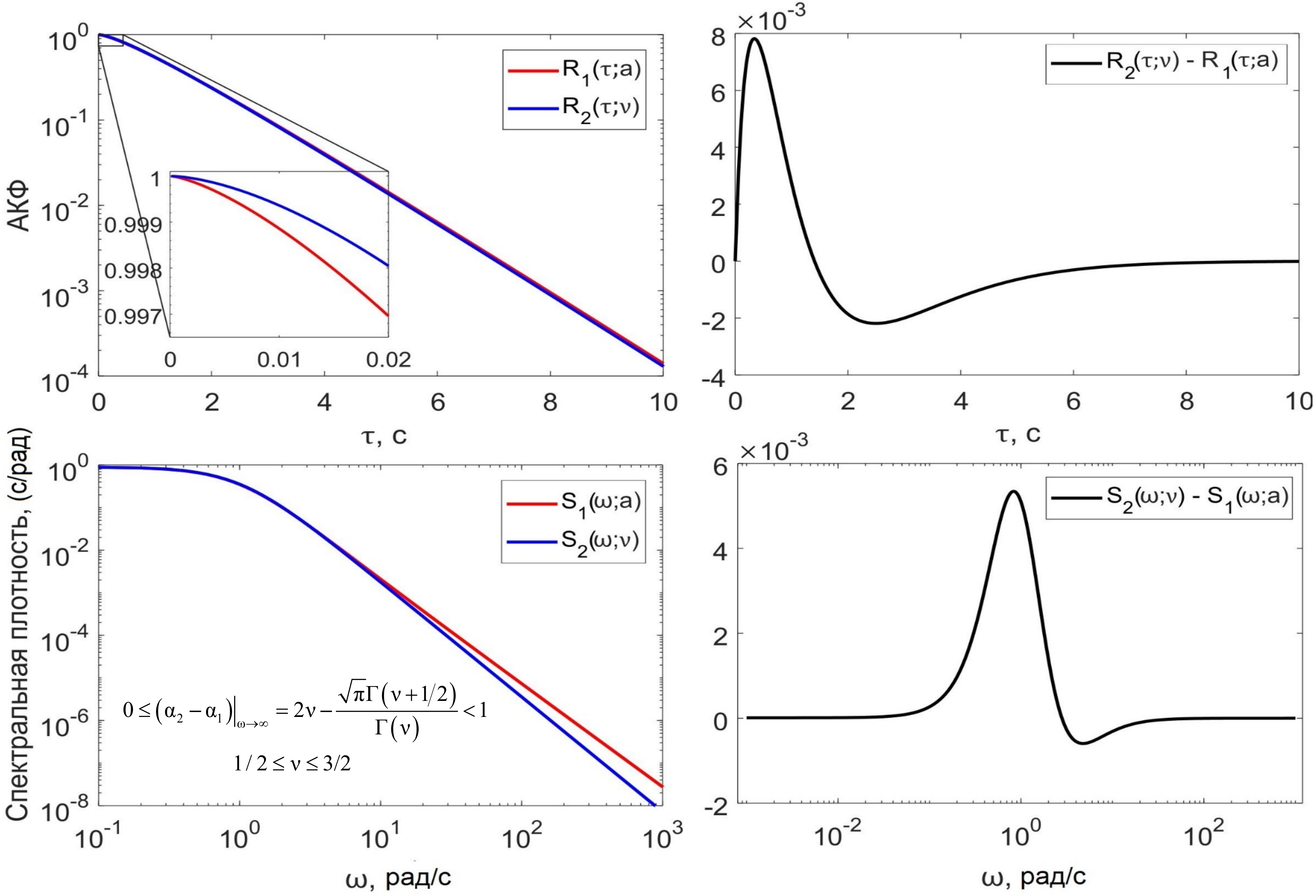
$$\frac{R_1(\tau_1; a)}{R_2(\tau_2; \nu)} - 1 \approx 8 \cdot 10^{-3}$$

## Distance between ACFs

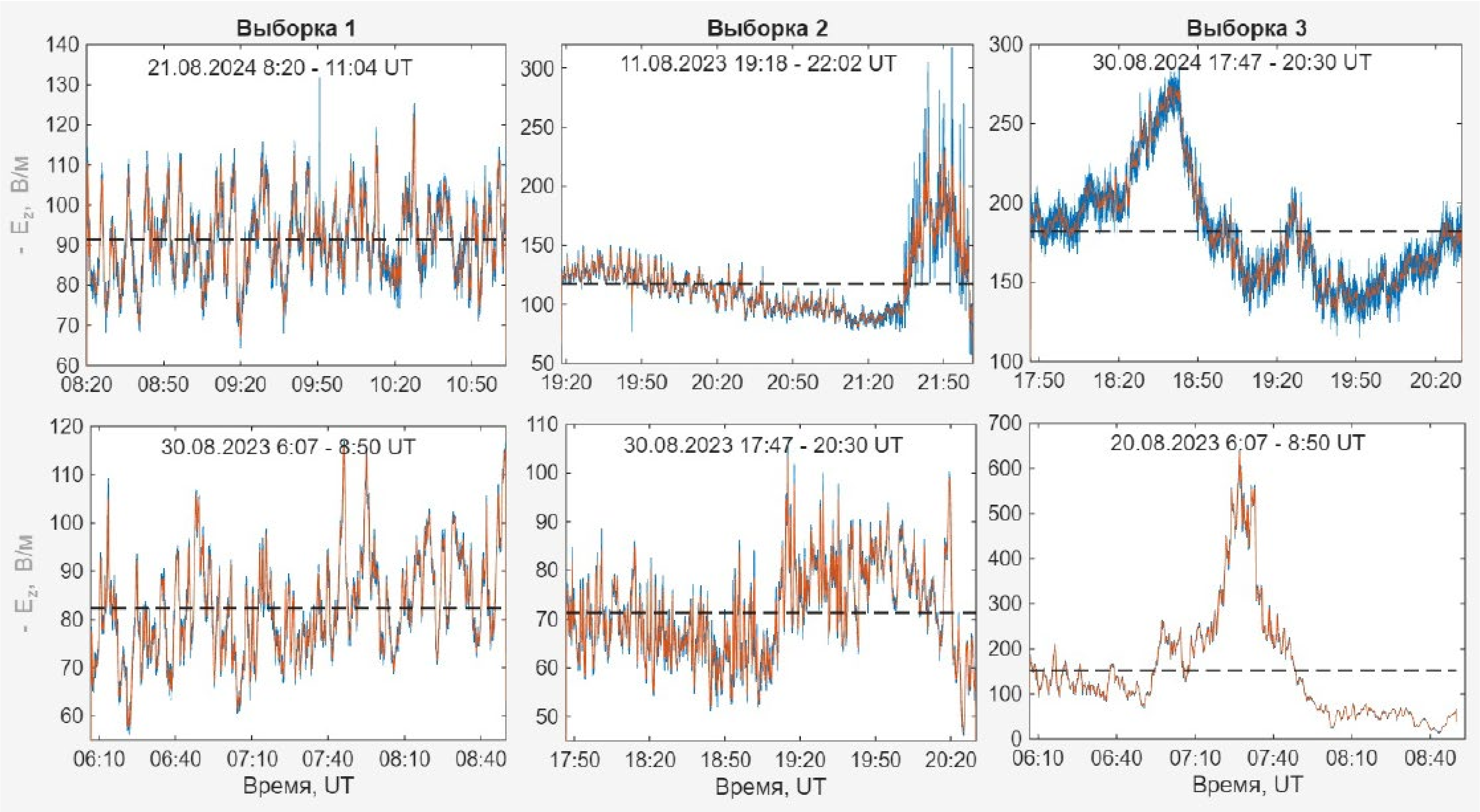
$$\sup_{x>0}\left|R_2(x; \nu) - R_1\left(x; \frac{\sqrt{\pi}\Gamma(\nu + 1/2)}{\Gamma(\nu)}\right)\right| < 2^{-7}$$

$$\sqrt{\int_0^\infty \left[R_2(x; \nu) - R_1\left(x; \frac{\sqrt{\pi}\Gamma(\nu + 1/2)}{\Gamma(\nu)}\right)\right]^2 dx} < 6.8 \cdot 10^{-3}$$

# Comparison of ACF-PSD models 1 and 2

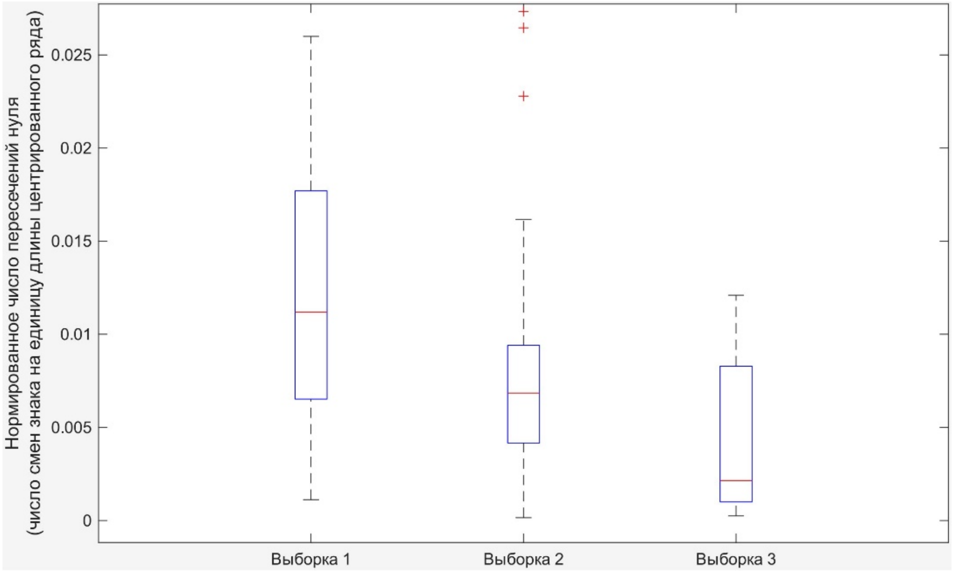


# Recordings $E_z$: separation by variance and time scale of ACF

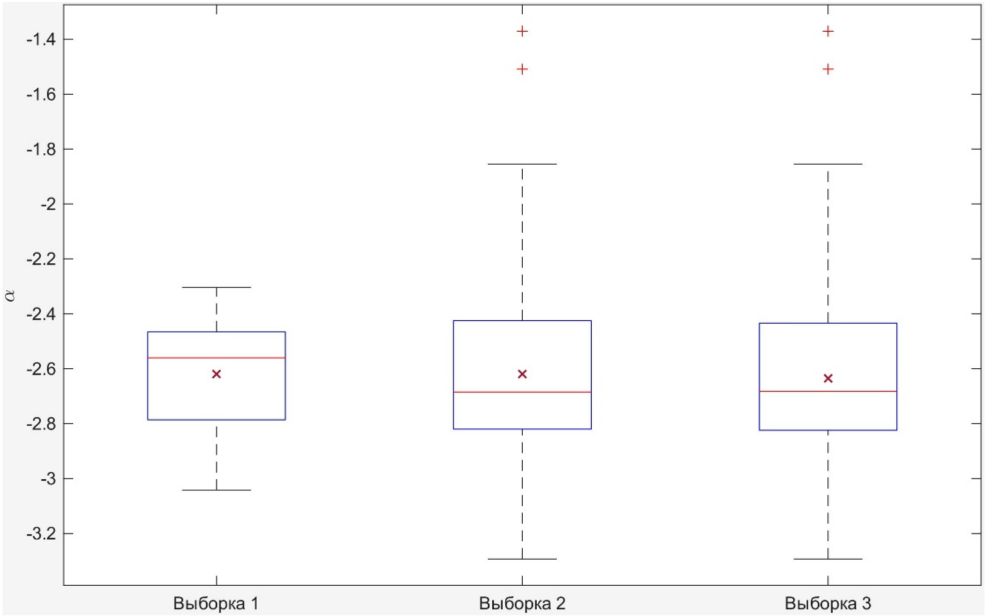


# Comparison of time series parameters and ACF-PSD models 1-3

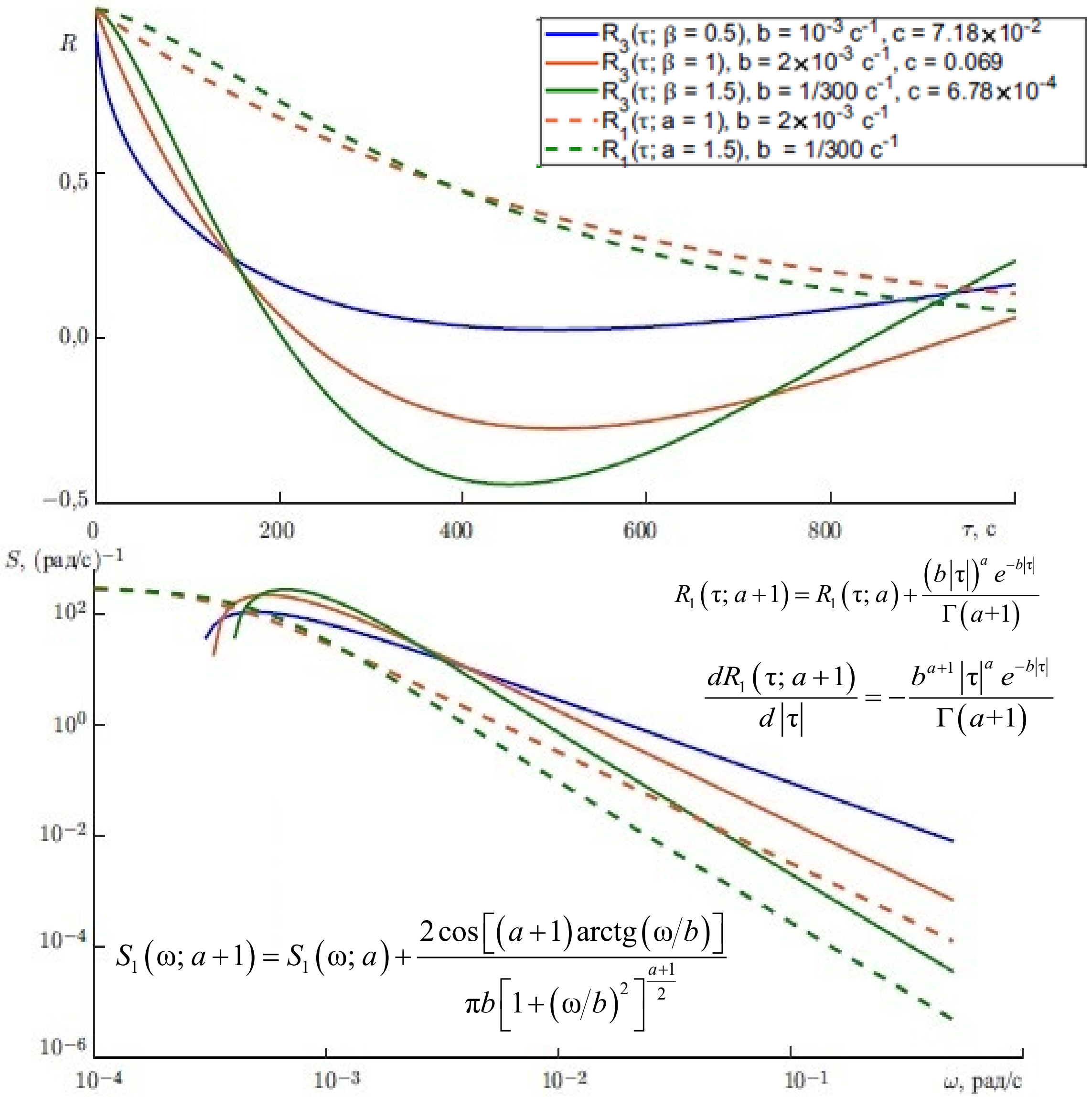
## Distribution of the density of the number of zeros of a normalized time series




## Distribution of the power-law approximation index of the PSD



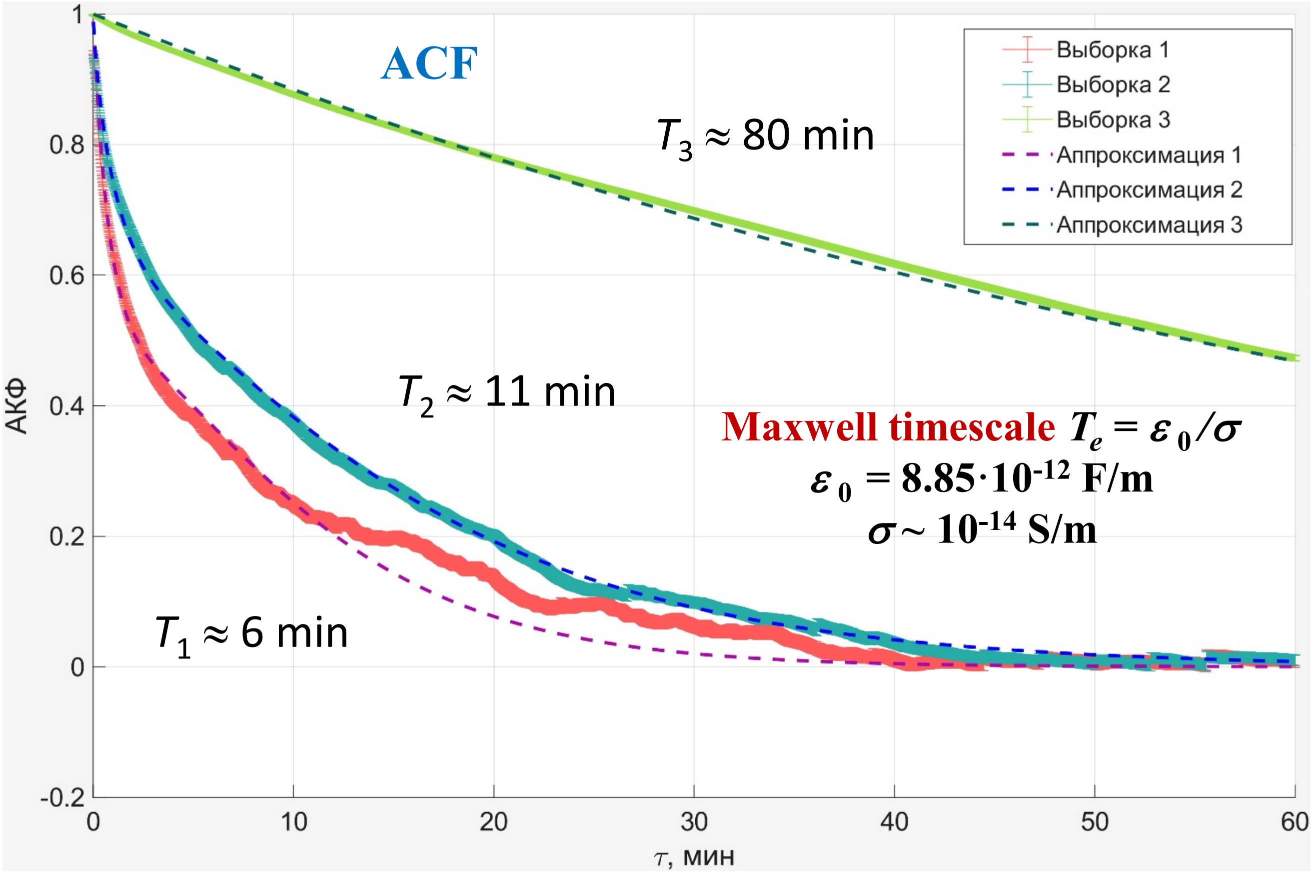

## ACF and PSD of models 1 and 3 for different parameter values

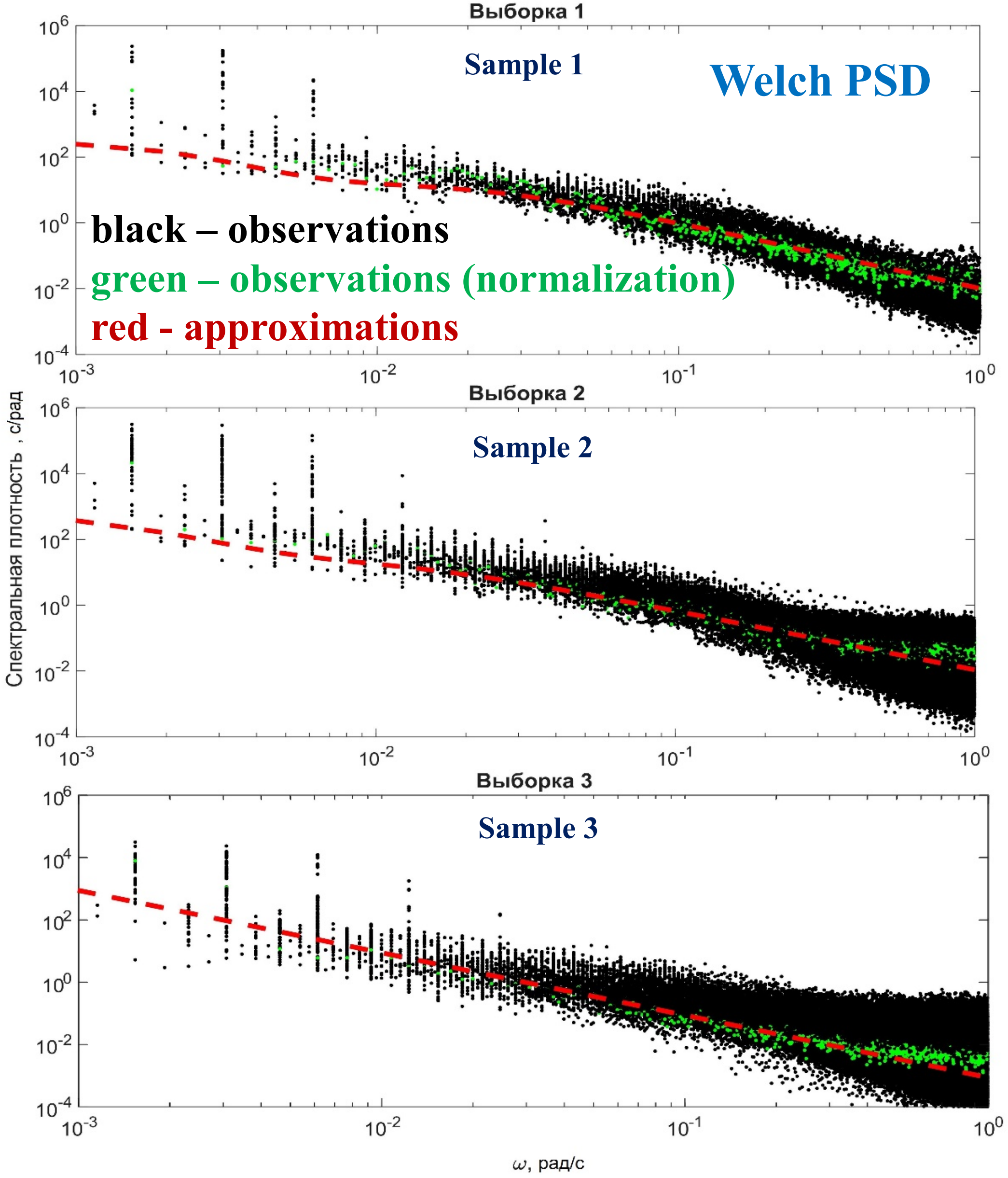


# Joint approximations of the ACF and the PSD of $E_z$ fluctuations

$$R(\tau) = c\frac{\Gamma(a_1, b_1|\tau|)}{\Gamma(a_1)} + (1-c)\frac{\Gamma(a_2, b_2|\tau|)}{\Gamma(a_2)}, \quad 0 < c \le 1$$

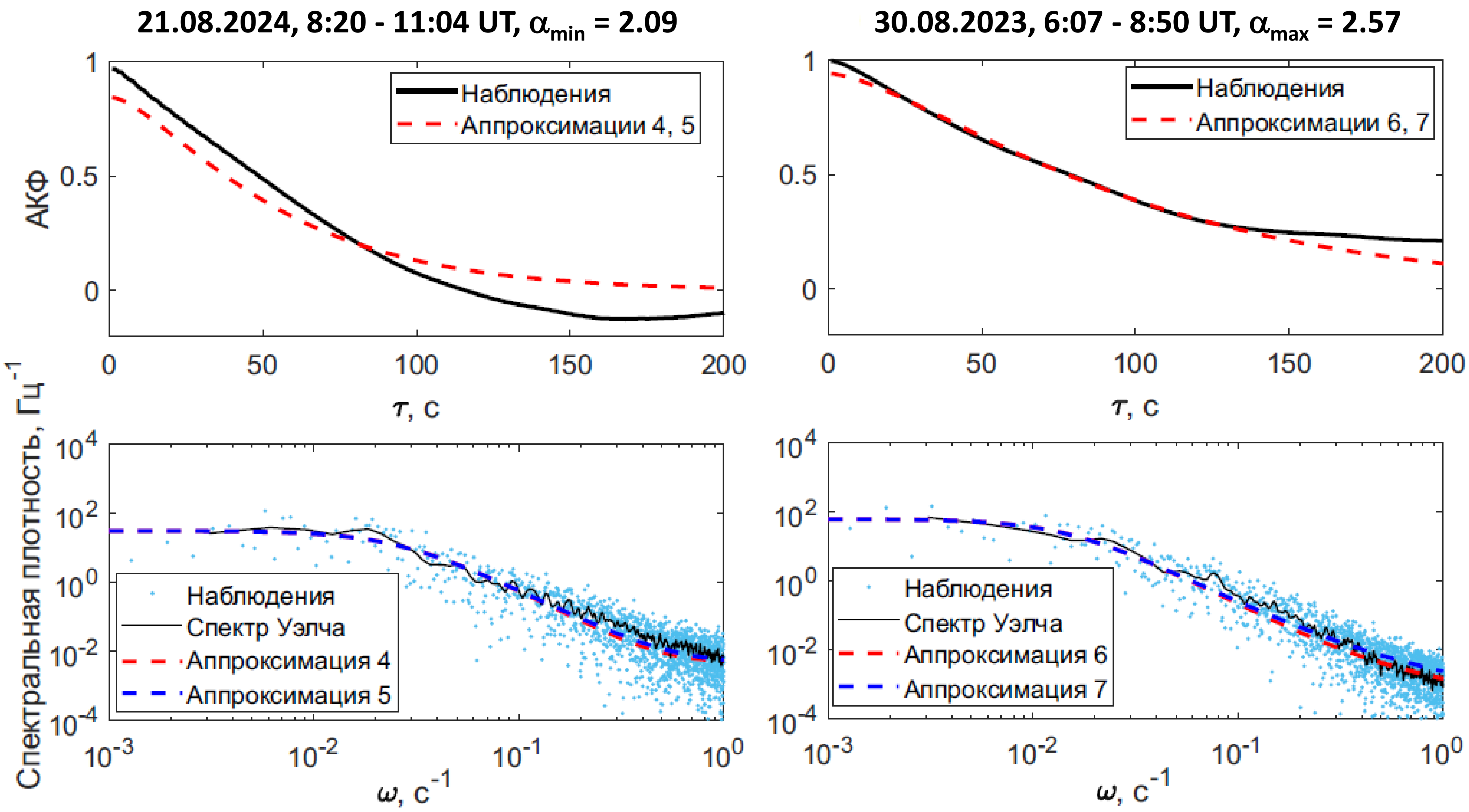
**Integral timescale** $$T = c\frac{a_1}{b_1} + (1-c)\frac{a_2}{b_2}$$



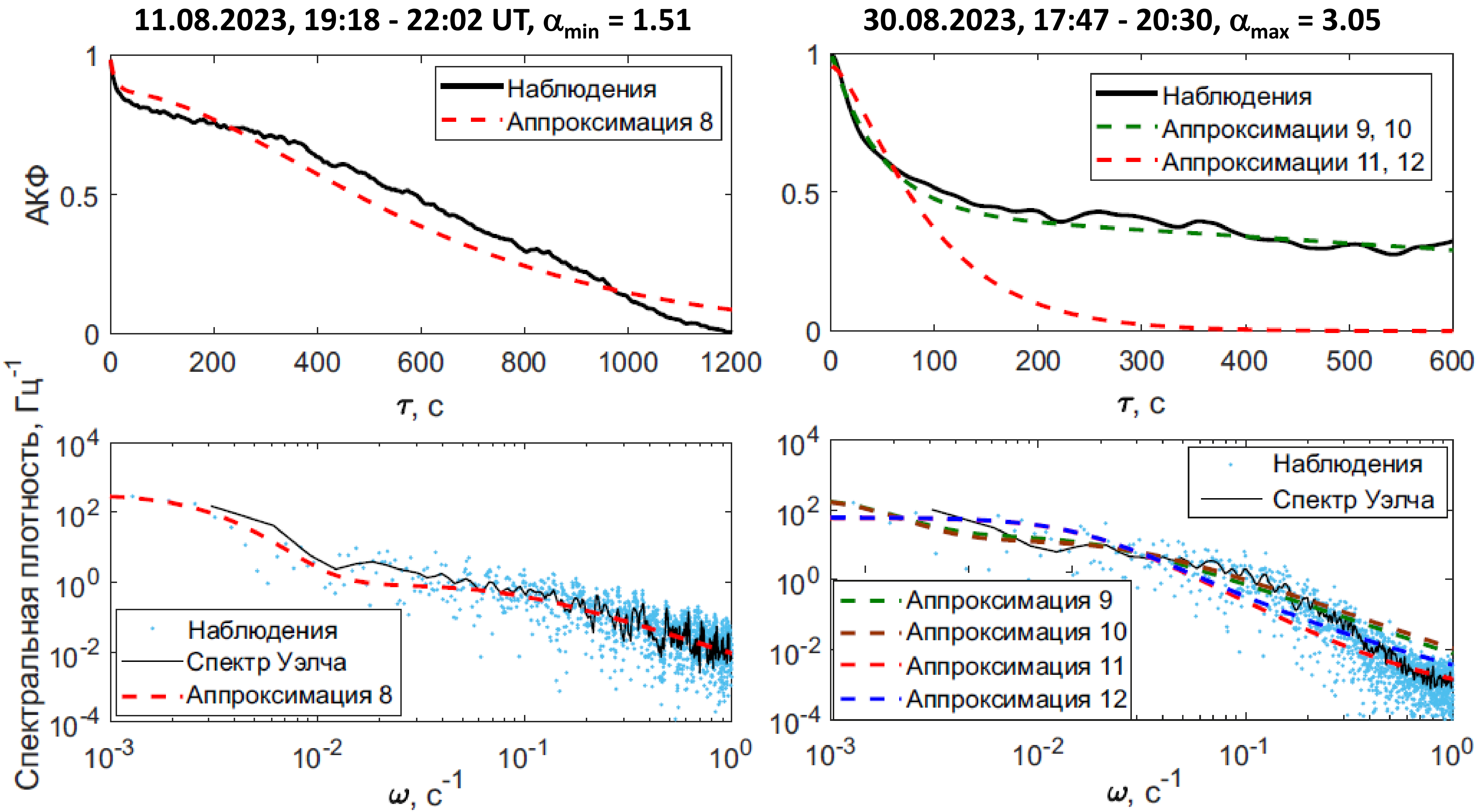
# Joint Levenberg-Marquardt approximations of the ACF and the PSD of $E_z$ fluctuations

## Sample 1



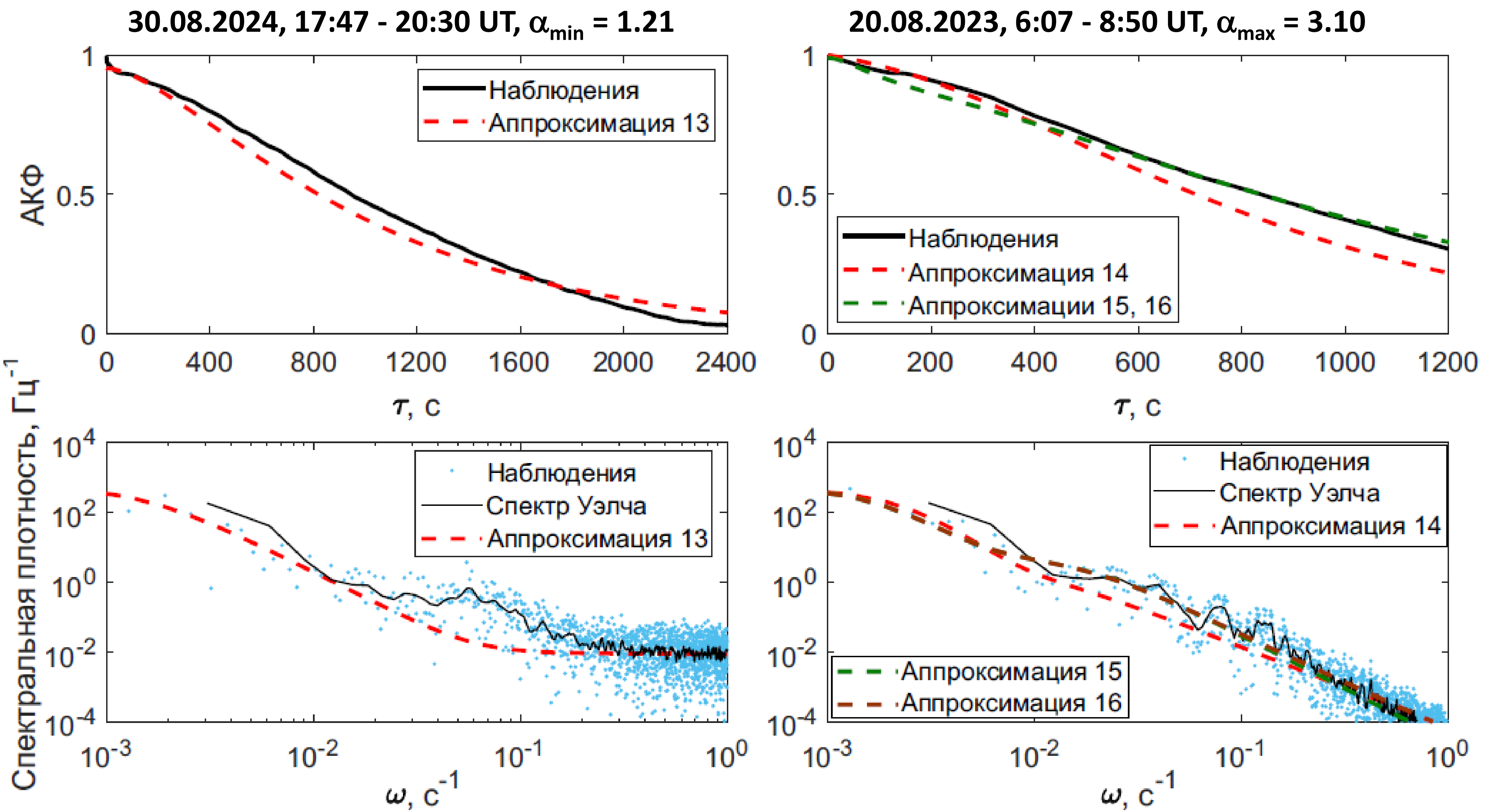
# Joint Levenberg-Marquardt approximations of the ACF and the PSD of $E_z$ fluctuations

## Sample 2



# Joint Levenberg-Marquardt approximations of the ACF and the PSD of $E_z$ fluctuations

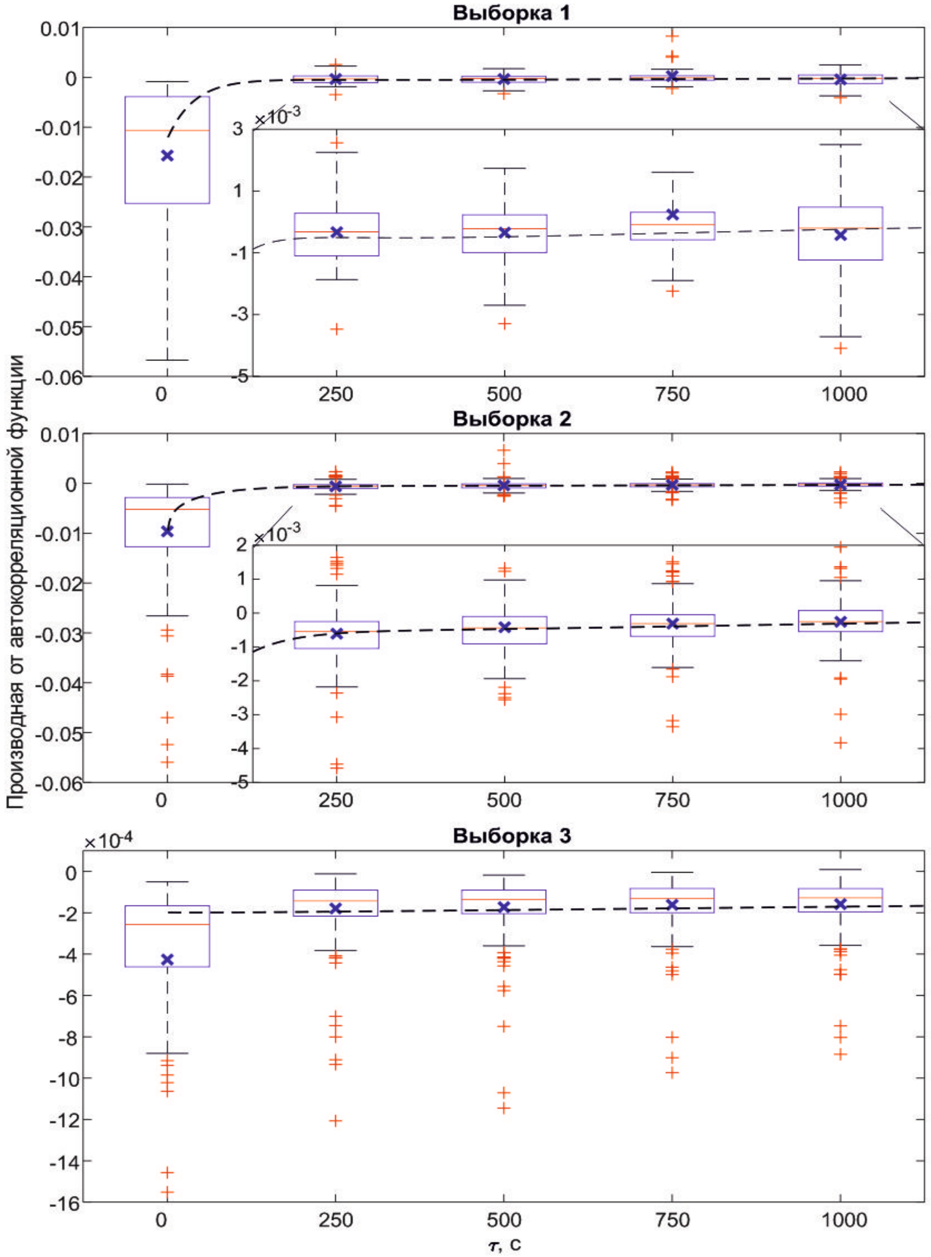
## Sample 3



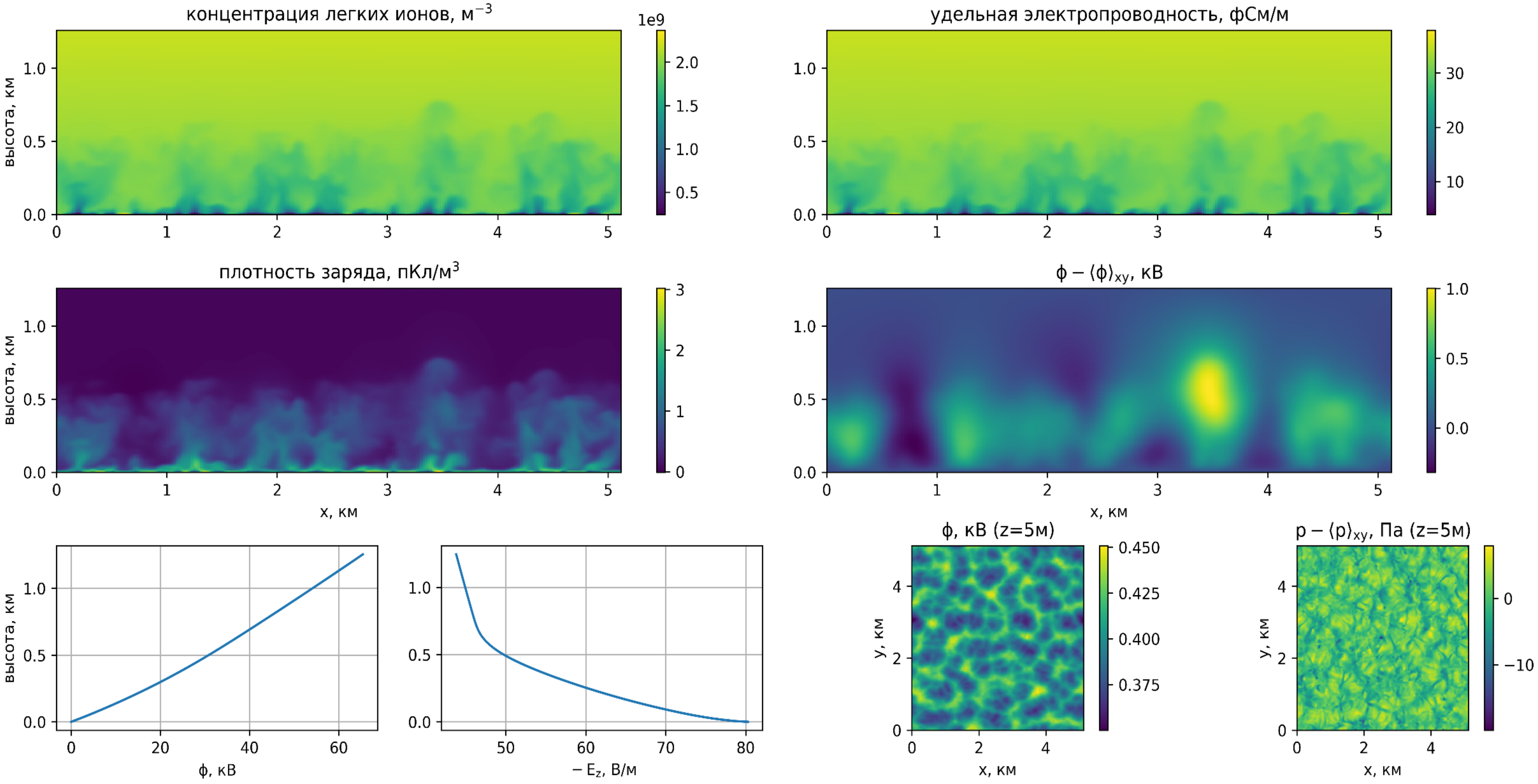
## Approximation parameters and deviation statistics

| № | Модель | Параметры аппроксимации | | | | | | | Оценка отклонения | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $a_1$ | $\nu_1$ | $b_1$, $c^{-1}$ | $a_2$ | $\nu_2$ | $b_2$, $c^{-1}$ | $c$ | $\Sigma$ | $r^2$ | $\Sigma_{\log}$ | $r^2_{\log}$ |
| 1 | $R_1$ | 1 | - | 1/40 | 2 | – | 1/360 | 0,5 | 0,248 | 0,953 | – | – |
| | $S_1$ | 1 | – | 1/40 | 2 | – | 1/360 | 0,5 | – | – | 0,201 | 0,941 |
| 2 | $R_1$ | 0,8 | – | 1/75 | 1,5 | – | 1/670 | 0,38 | 0,012 | 0,990 | – | – |
| | $S_1$ | 0,8 | – | 1/75 | 1,5 | – | 1/670 | 0,38 | – | – | 0,260 | 0,911 |
| 3 | $R_1$ | 1,5 | – | 1/2 400 | 1 | – | 1/4 800 | 0,05 | 0,009 | 0,996 | – | – |
| | $S_1$ | 1,5 | – | 1/2 400 | 1 | – | 1/4 800 | 0,05 | – | – | 0,013 | 0,997 |
| 4 | $R_2$ | – | 0,187 | 10 | – | 1 | 0,027 | 0,05 | 0,105 | 0,907 | – | – |
| | $S_2$ | – | 0,187 | 10 | – | 1 | 0,027 | 0,155 | – | – | 0,782 | 0,684 |
| 5 | $R_1$ | 0,473 | – | 10 | $\pi/2$ | – | 0,027 | 0,155 | 0,107 | 0,904 | – | – |
| | $S_1$ | 0,473 | – | 10 | $\pi/2$ | – | 0,027 | 0,155 | – | – | 0,725 | 0,729 |
| 6 | $R_2$ | – | 0,011 | 0,01 | – | 1 | 0,015 | 0,066 | 0,040 | 0,974 | – | – |
| | $S_2$ | – | 0,011 | 0,01 | – | 1 | 0,015 | 0,066 | – | – | 0,756 | 0,824 |
| 7 | $R_1$ | 0,033 | – | 0,01 | $\pi/2$ | – | 0,015 | 0,066 | 0,039 | 0,975 | – | – |
| | $S_1$ | 0,033 | – | 0,01 | $\pi/2$ | - | 0,015 | 0,066 | – | – | 0,496 | 0,924 |
| 8 | $R_2$ | – | 0,418 | 0,085 | – | 2 | $3{,}7 \cdot 10^{-3}$ | 0,129 | 0,061 | 0,948 | – | – |
| | $S_2$ | – | 0,418 | 0,085 | – | 2 | $3{,}7 \cdot 10^{-3}$ | 0,129 | – | – | 0,691 | 0,845 |
| 9 | $R_2$ | – | 0,5 | 0,02 | – | 1,5 | $1{,}7 \cdot 10^{-3}$ | 0,6 | 0,029 | 0,954 | – | – |
| | $S_2$ | – | 0,5 | 0,02 | – | 1,5 | $1{,}7 \cdot 10^{-3}$ | 0,6 | – | – | 1,081 | 0,700 |
| 10 | $R_1$ | 1 | – | 0,02 | 2 | – | $1{,}7 \cdot 10^{-3}$ | 0,6 | 0,029 | 0,954 | – | – |
| | $S_1$ | 1 | – | 0,02 | 2 | – | $1{,}7 \cdot 10^{-3}$ | 0,6 | – | – | 1,081 | 0,700 |
| 11 | $R_2$ | – | 0,013 | 0,01 | – | 1 | 0,016 | 0,052 | 0,075 | 0,695 | – | – |
| | $S_2$ | - | 0,013 | 0,01 | – | 1 | 0,016 | 0,052 | – | – | 0,884 | 0,800 |
| 12 | $R_1$ | 0,040 | – | 0,01 | $\pi/2$ | - | 0,016 | 0,052 | 0,073 | 0,696 | – | – |
| | $S_1$ | 0,040 | – | 0,01 | $\pi/2$ | – | 0,016 | 0,052 | – | – | 0,688 | 0,879 |
| 13 | $R_2$ | – | 1 | 5,086 | – | 1 | $1{,}5 \cdot 10^{-3}$ | 0,046 | 0,053 | 0,969 | – | – |
| | $S_2$ | – | 1 | 5,086 | – | 1 | $1{,}5 \cdot 10^{-3}$ | 0,046 | – | – | 0,555 | 0,613 |
| 14 | $R_2$ | – | 0,659 | 0,011 | – | 2 | $2{,}8 \cdot 10^{-3}$ | 0,032 | 0,069 | 0,893 | – | – |
| | $S_2$ | – | 0,659 | 0,011 | – | 2 | $2{,}8 \cdot 10^{-3}$ | 0,032 | – | – | 0,870 | 0,904 |
| 15 | $R_2$ | – | 1,031 | 0,018 | – | 1,5 | $1{,}8 \cdot 10^{-3}$ | 0,1 | 0,025 | 0,986 | – | – |
| | $S_2$ | – | 1,031 | 0,018 | – | 1,5 | $1{,}8 \cdot 10^{-3}$ | 0,1 | – | – | 0,964 | 0,882 |
| 16 | $R_1$ | 1,6 | – | 0,018 | 2 | – | $1{,}8 \cdot 10^{-3}$ | 0,1 | 0,025 | 0,986 | – | – |
| | $S_1$ | 1,6 | – | 0,018 | 2 | – | $1{,}8 \cdot 10^{-3}$ | 0,1 | – | – | 0,709 | 0,936 |

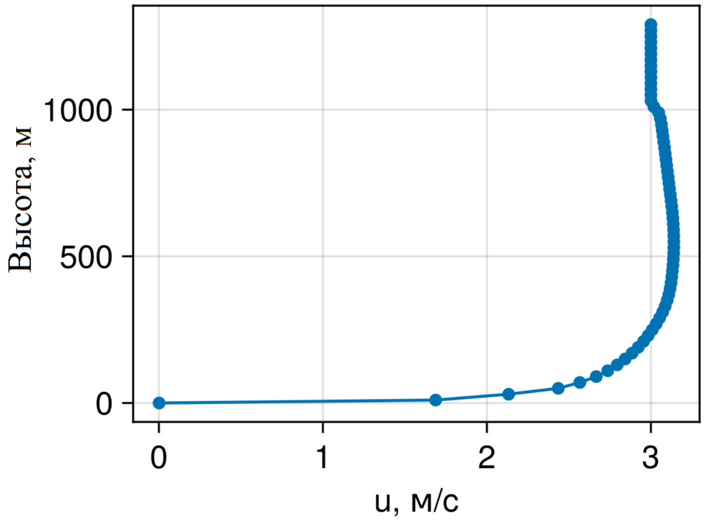
## Statistics of ACF difference derivatives and derivatives of approximating functions

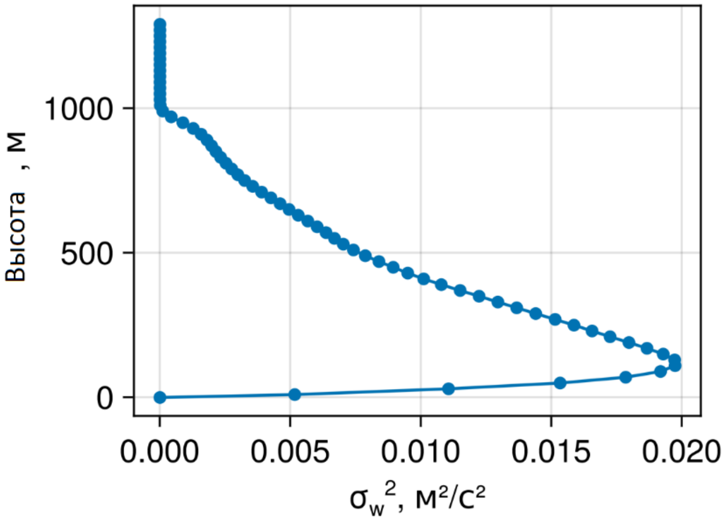


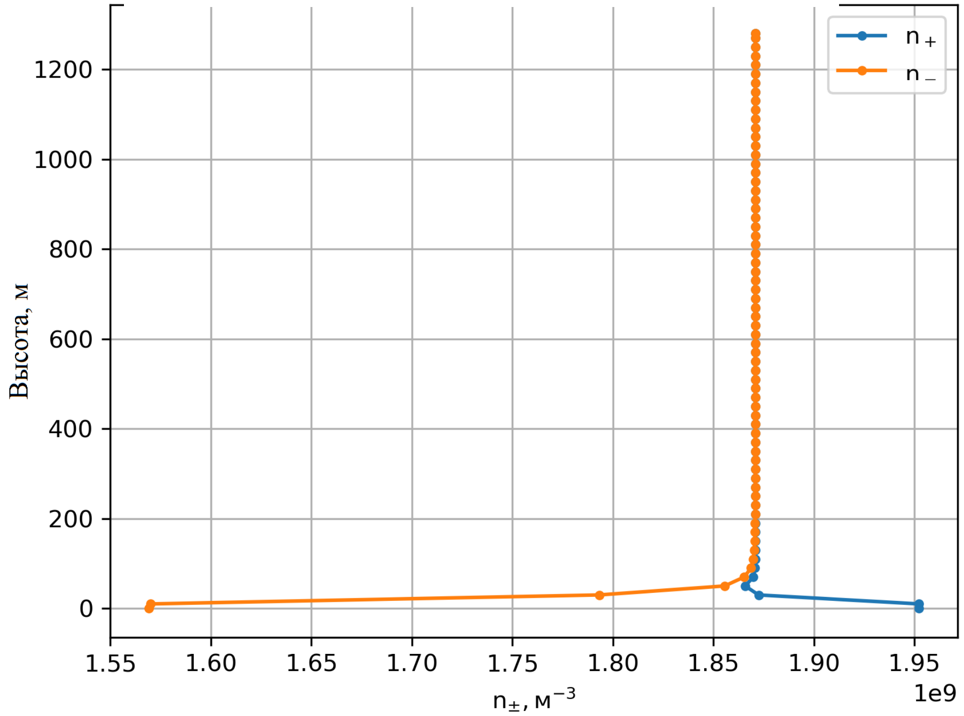
# Adaptation of the electrodynamics module in PALM for large-eddy simulation

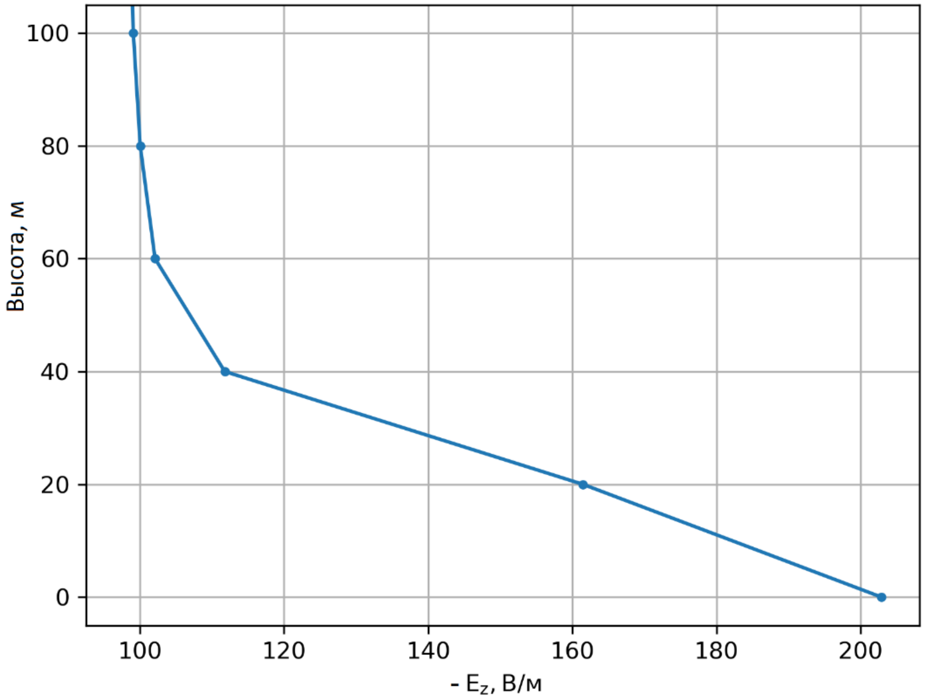


# Average LES vertical profiles

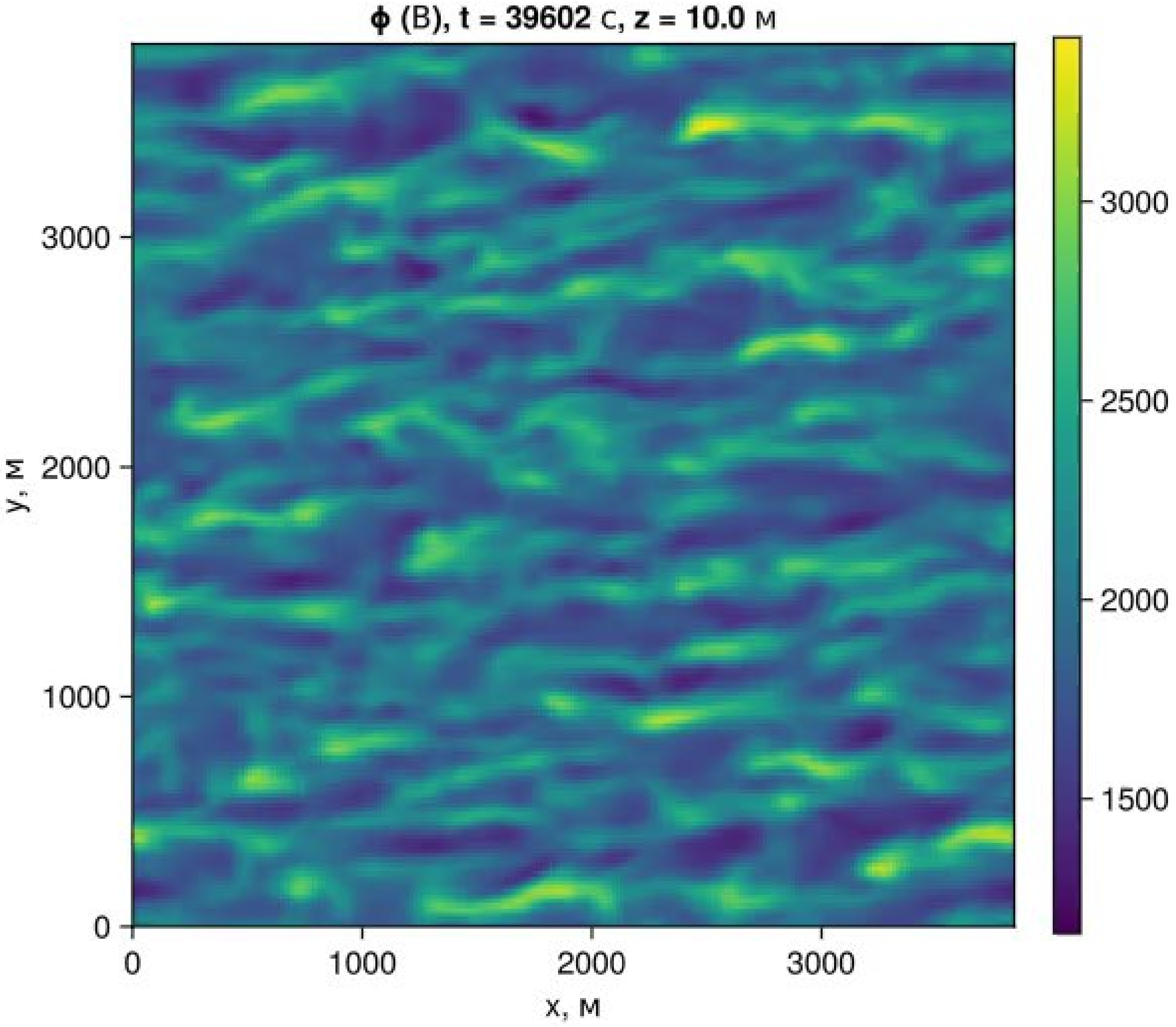


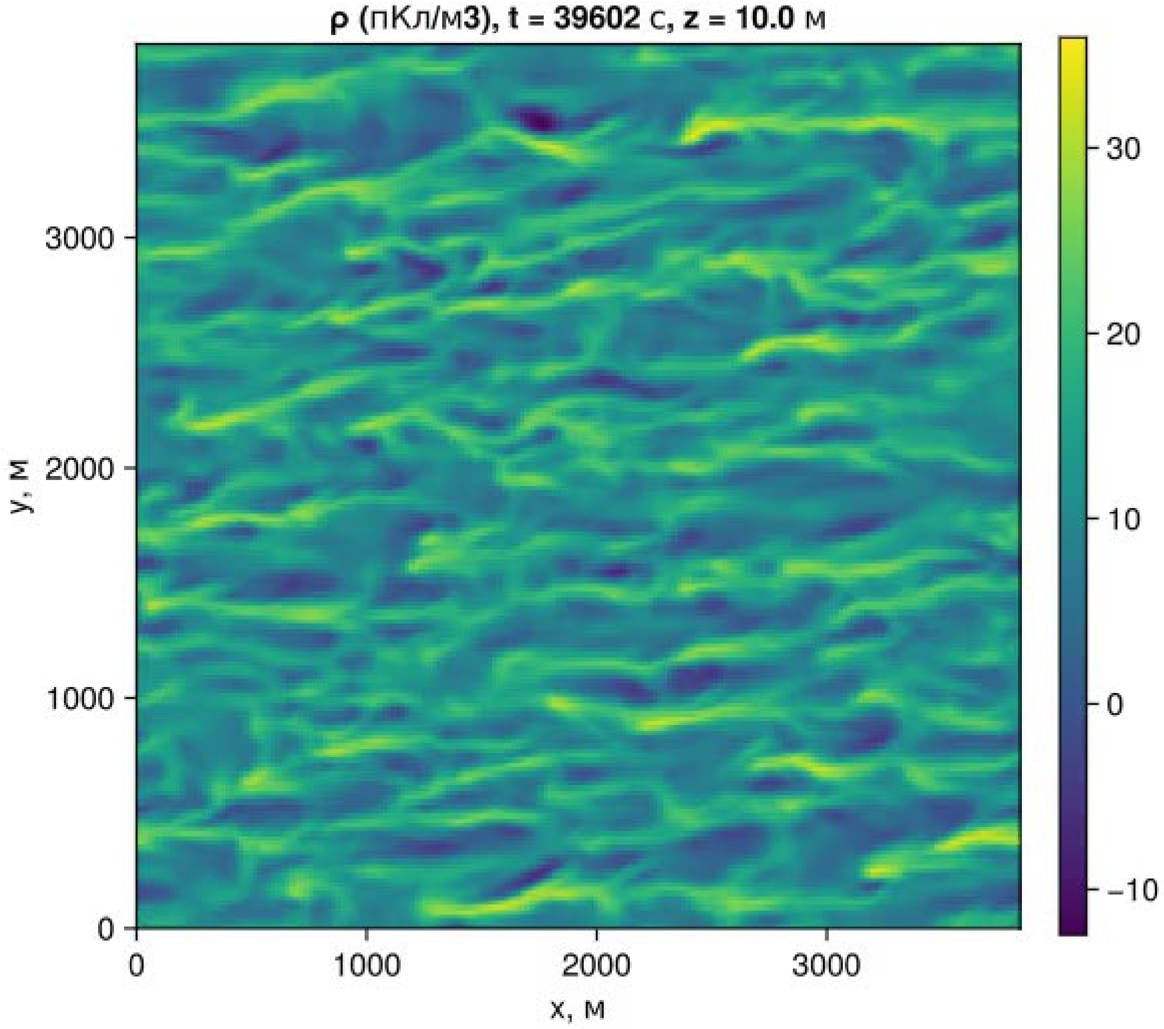



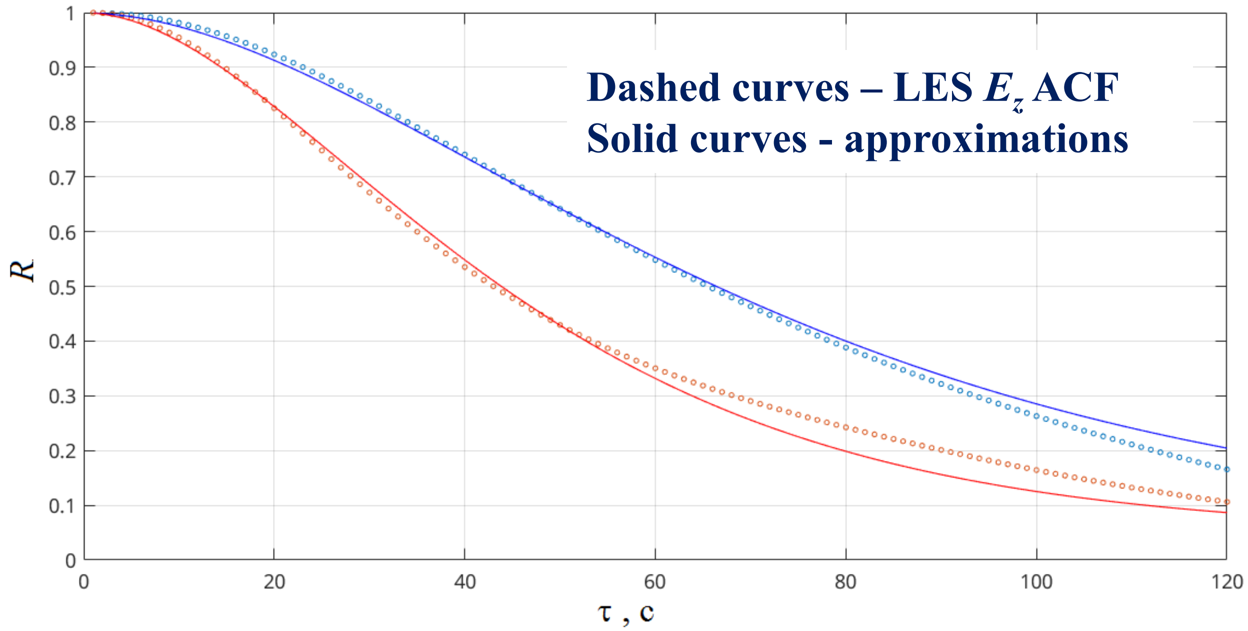



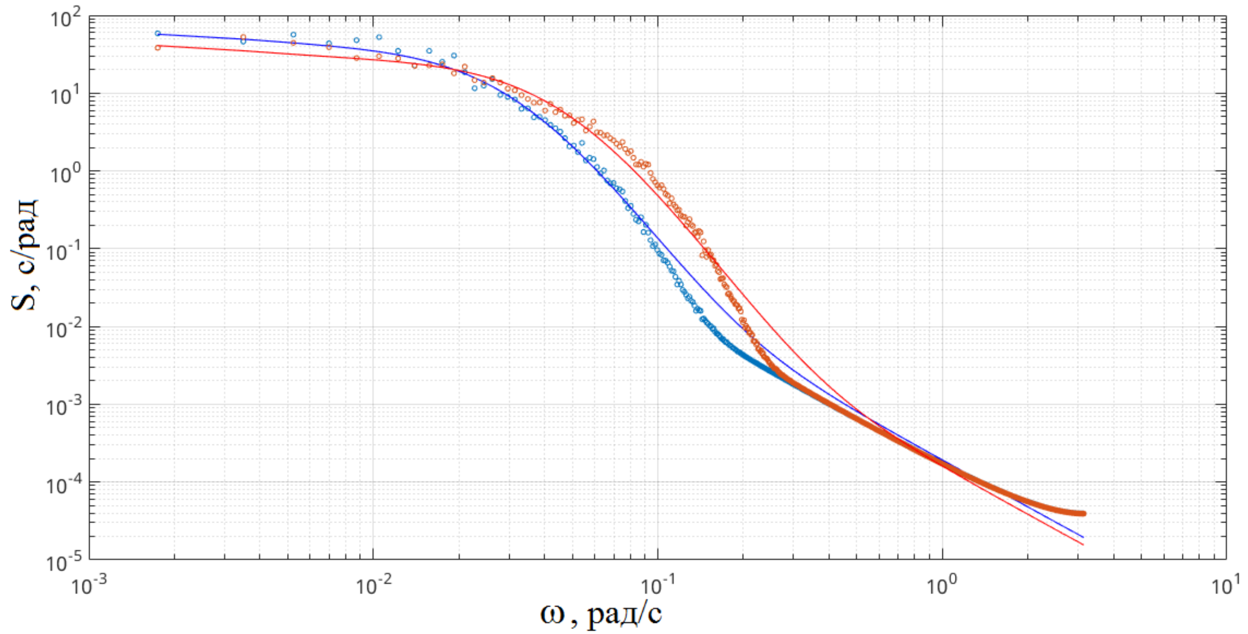
# LES dynamics of electric space charge and potential difference



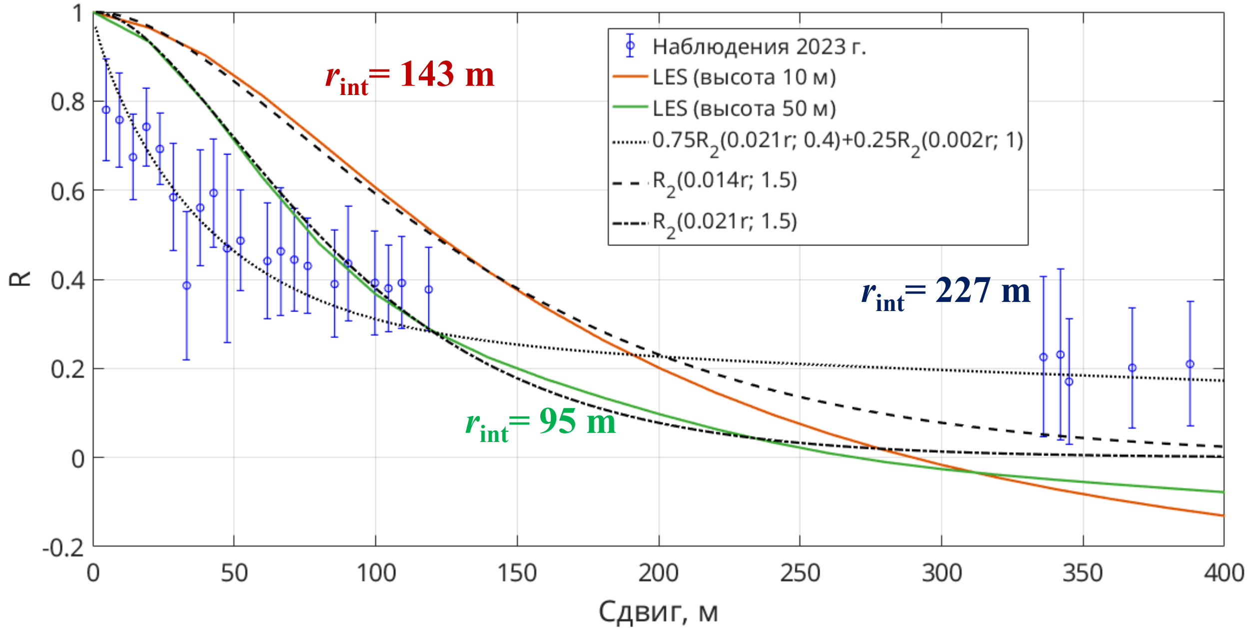
# Joint approximations of the ACF and the PSD of $E_z$ LES fluctuations




$z = 10$ m
$c = 0.9$
$b_1 = 0.0325$ s$^{-1}$
$b_2 = 0.003$ s$^{-1}$
$\nu_1 = 2$
$\nu_2 = 0.5$
$\sigma^2 = 208$ (V/m)$^2$
$T = 99$ s

$z = 50$ m
$c = 0.92$
$b_1 = 0.049$ s$^{-1}$
$b_2 = 0.003$ s$^{-1}$
$\nu_1 = 2$
$\nu_2 = 0.5$
$\sigma^2 = 30$ (V/m)$^2$
$T = 71$ s



# Conclusions

- Using nonlinear regression methods, the parameters of the models of joint approximations of the ACF and the PSD of observed and LES turbulent fluctuations $E_z$ were determined, which have a power-law asymptotic behavior in the high-frequency region.

- Models that differ in ACF by less than 1% in the Chebyshev and $L^2$ metrics describe random processes with different local properties and have a power-law asymptotic behavior of the spectral density in the high-frequency region $S \propto f^{-\alpha}$ with different indices, the difference of which can approach 1.

- New approximate relationships are defined between incomplete gamma functions with parameter $a \in [1, 2]$ and Macdonald functions with parameter $\nu \in [0.5, 1.5]$, which become exact on the corresponding boundaries of the specified segments.

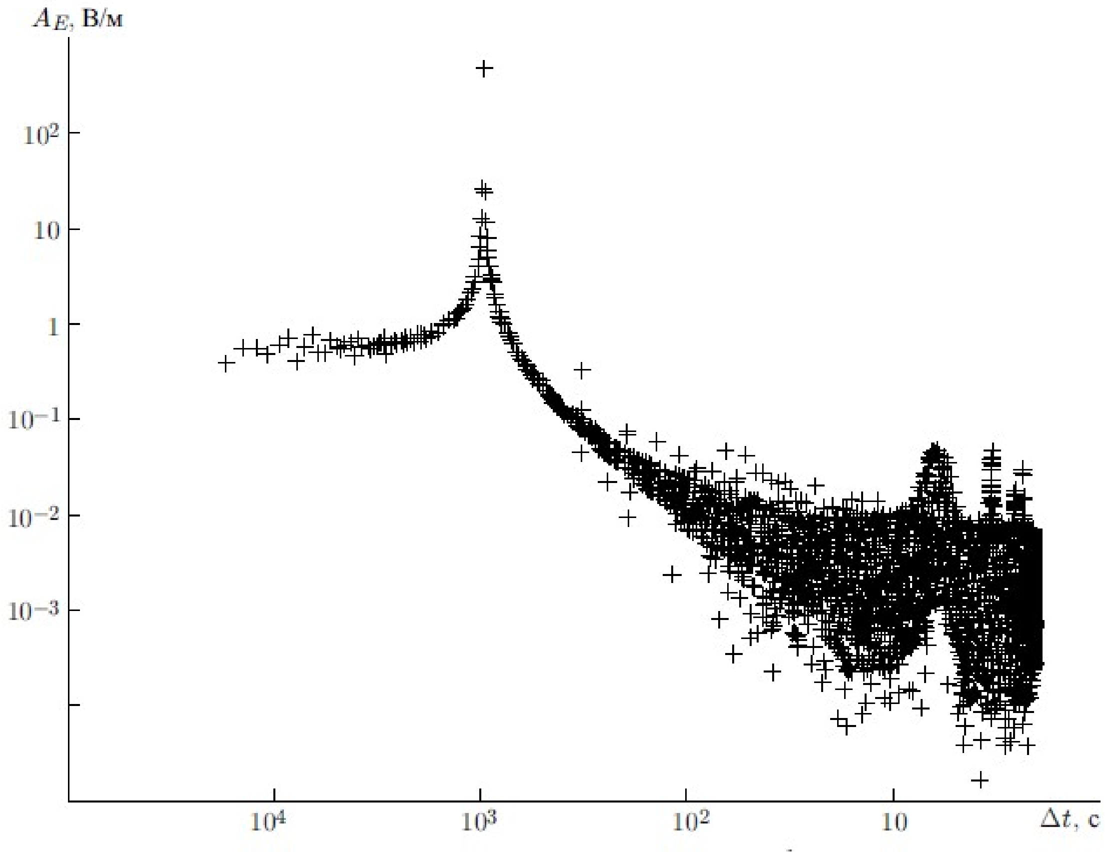
- When analyzing time intervals of finite length, significantly different ACFs may correspond to similar α indices in the high-frequency asymptotics of the spectral density.

- In the samples under consideration, at least 95% of the time series segments are characterized by an ACF that does not have a second derivative at zero, and the $E_z$ time series with a resolution of 1 s are not differentiable in the root mean square.

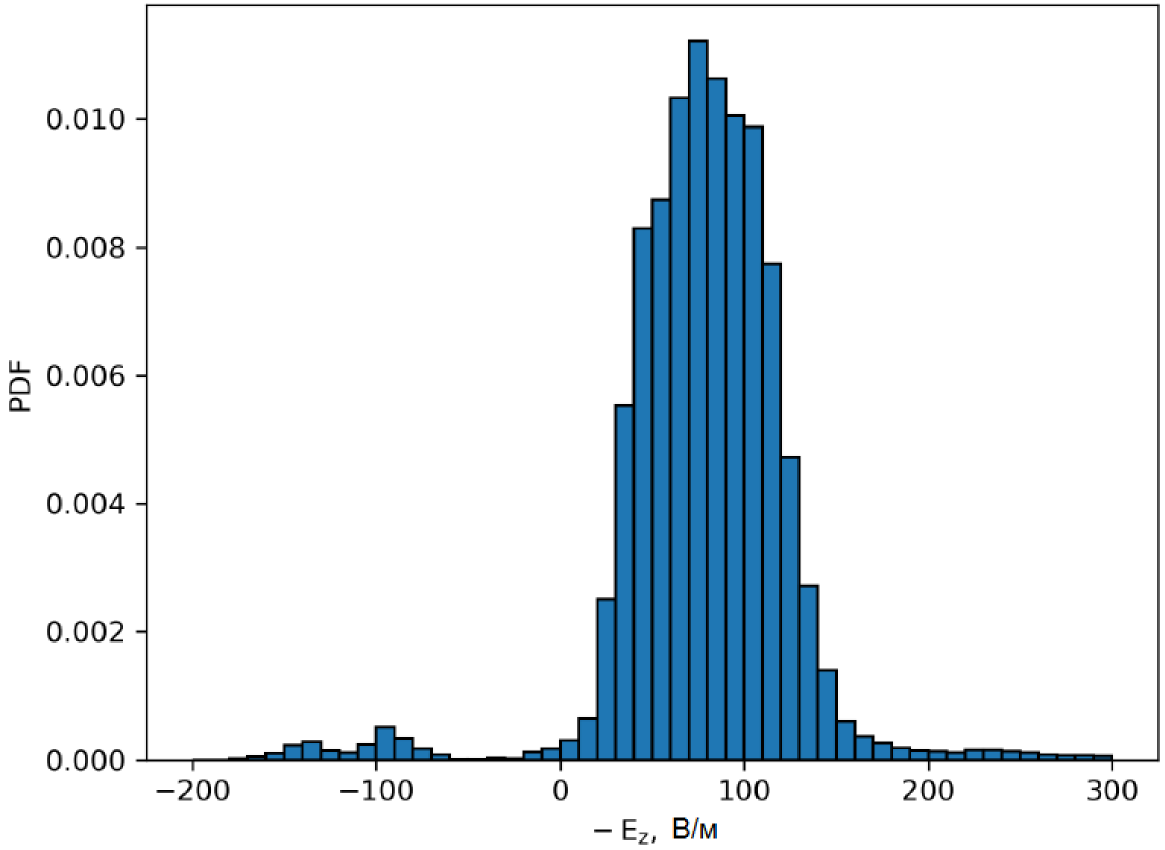
The developed methods and obtained results can be applied to the analysis of time series of physical quantities of any nature, the spectral density of fluctuations of which has a power-law asymptotic behavior in a limited frequency range and saturates in the low-frequency region.

Работа выполнена при частичном финансировании РНФ (проект № 22-17-00053-П).

# СПАСИБО ЗА ВНИМАНИЕ!




**Спектральная амплитуда в периодной шкале стендовой регистрограммы калибровочного синусоидального сигнала с частотой $10^{-3}$ Гц и амплитудой 500 В/м**




**Распределения 3.58· $10^5$ измеренных с частотой 1 Гц величин вертикальной компоненты атмосферного электрического поля в 2023 и 2024**