A Lag between Temperature and Atmospheric CO₂ Concentration
Based on a Simple Coupled Model of Climate and the Carbon Cycle

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Determination of the role of natural and anthropogenic factors in the modern climate change is one of the key problems of the 21st century. The objective of this work is to estimate the degree of the cause-and-effect correlations in the Earth system (ES) that can be judged from the time shifts in the time series of data, in particular, in the data on the temperature and CO₂ concentration in the atmosphere.

The data of ice cores used to reconstruct climate changes in the Pleistocene, in particular, the glacia tion cycles, reveal a general time lag of the carbon dioxide concentration in the atmosphere q relative to the variations in the surface temperature T (see, for example, [1–3]).

Frequently, the general lag of q relative to T revealed from the paleo-data is an argument against the statement that the modern global warming is caused by the greenhouse effect of the anthropogenic increase in q. The lag of q relative to T was found in [4] from the analysis of observations in 1980–2010. This was the basis to conclude that the modern emissions of greenhouse gases are not the cause of the modern climate warming.

It is worth noting that glaciation cycles are related to variations in the parameters of the Earth’s orbit (the so-called Milankovitch cycles) with characteristic periods of approximately 100 000, 40 000, and 20 000 years. Climate changes (in particular, the temperature) occur due to variations in the orbit, and the temperature variations facilitate the variations in the concentration of greenhouse gases in the atmosphere. The latter, in turn, influences the temperature variations. A similar effect can be also manifested if caused by the other radiative components.

Variations in the CO₂ concentration in the atmosphere can also occur due to the variations in the solubility of the gas in seawater caused by temperature variations. During warm periods, CO₂ is released into the atmosphere from the ocean. In this relation, the increase in the CO₂ concentration in the atmosphere is sometimes interpreted as a consequence but not a cause of global warming occurring.

In this work we show that the found mutual lags between the variations in the temperature and carbon dioxide concentrations in the atmosphere do not contradict the conclusions that the key role in the modern climate changes belongs to the anthropogenic greenhouse effect [1, 5]. In particular, it was found that the dependence of CO₂ dissolubility in the ocean on temperature does not have any principal influence on the time lag between q and T when external forcing is applied to the system.

We consider a globally average climate model with the carbon cycle, which takes into account the generally accepted influence mechanisms of radiative perturbing forcing (including the greenhouse one) on the climate state and the interaction between the climate and the carbon cycle:

\[ \frac{dq}{dt} = E(t) - F_{oc} - F_{land}, \]  \hspace{1cm} (1)

\[ \frac{dD}{dt} = F_{oc}, \]  \hspace{1cm} (2)

\[ \frac{d(M_b + M_s)}{dt} = F_{land}, \]  \hspace{1cm} (3)

\[ C \frac{dT}{dt} = R \ln \left(1 + \frac{q}{q_0}\right) - \lambda T + R_f(t). \]  \hspace{1cm} (4)

Here, q is the deviation of the CO₂ concentration in the atmosphere from the initial (preindustrial) value \( q_0 = 590 \text{ Gt of C} \) (which corresponds to a concentration of 278 ppm); D is the corresponding deviation of carbon resources in the ocean; \( M_b \) and \( M_s \) are the deviations of carbon stocks in the vegetation and soil, respectively; T is the temperature deviation; \( E(t) \) are external (including anthropogenic) CO₂ emissions into the atmosphere; \( F_{oc} \) is the CO₂ flux from the atmosphere to the ocean; \( F_{land} \) is the CO₂ flux from the atmosphere to the terrestrial ecosystems; \( C = 10^9 \text{ J m}^{-2} \text{ K}^{-1} \) is the thermal capacity of a unit square of the Earth’s
The CO₂ flux from the atmosphere is calculated using the Bacastow model with an evasion factor depending on the temperature and CO₂ concentration in the atmosphere [6, 7]:

\[ F_{oc} = F_0 \chi(T) \left( q - \zeta(T, q) \frac{R_T}{D_b} \right), \]

where \( F_0 \) is the coefficient determined from the observations in the 20th century [1] with the appropriate boundary conditions for \( q \) and \( T \left( F_0 = (2.5-4.5) \times 10^{-2} \text{ Gt C yr}^{-1} \right) \); \( \chi(T) \) is the characteristic of CO₂ solubility in seawater; \( \zeta \) is the buffer factor; \( D_b = 1.5 \times 10^5 \text{ Gt C} \).

The CO₂ flux from the atmosphere to the terrestrial ecosystems is calculated using the following scheme [8]

\[ F_{land} = P - BR - SR, \]

where \( P \) is consumption of CO₂ by means of photosynthesis and \( BR \) and \( SR \) are emissions of CO₂ due to autotrophic and heterotrophic respiration (respiration of vegetation and soil), respectively. In turn,

\[ P = A_p g_f(q) \theta_p^T, \]
\[ BR = A_R M_s \theta_b^T, \]
\[ SR = A_R M_s \theta_s^T, \]

where \( A_p = 0.1818 \text{ Gt C yr}^{-1}; A_R = 0.0909 \text{ yr}^{-1}; g(q) \) is a function characterizing fertilization of terrestrial vegetation by the carbon dioxide of the atmosphere; \( \theta_p = 1.04, \theta_b = 1.08, \theta_s = 1.09 \) [9].

We performed numerical experiments using this model with different values of input parameters in the equations and different types of periodical forcing:

1. periodical (sinusoidal) emissions of CO₂ into the atmosphere \( E(t) = E_0 \sin(\omega_E t) \) and zero RF \( R_F(t) = 0 \);

2. periodical (sinusoidal) RF \( R_F(t) = R_0 \sin(\omega_R t) \) and zero emissions of CO₂ into the atmosphere \( E(t) = 0 \) (these experiments are similar to the experiments analyzed in [10]);

3. periodical (sinusoidal) RF \( R_F(t) = R_0 \sin(\omega_R t) \) and exponential emissions of CO₂ into the atmosphere \( E(t) = E_0 e^{t/\tau_E} \).

We model the situation of the last decades of the 20th century using this type of forcing.

We analyzed the mutual time lag (phase shift) \( \Delta T_q \) between the variations in \( T \) and \( q \). The value of \( \Delta T_q \) was determined from the maximum of the correlation coefficient with a time shift between the time series of \( T \) and \( q \) similar to [2, 4], and also additionally between the time series of their first differences \((q(i + 1) - q(i))\) and \((T(i + 1) - T(i))\) (year—year). The characteristic values of the maximum correlation coefficient in cases (1) and (2) are not smaller than 0.95. In case (3), these values are smaller but also statistically reliable.

In case (1), there is a lag of \( T \) relative to \( q \) for any values of parameters \( \Delta T_q < 0 \) (Fig. 1). If the period of external forcing increases \( P = \frac{2\pi}{\omega_R} \), the lag \( \Delta T_q \) asymptotically tends to a value \( 23 \) years characterizing the thermal inertia of the ES.

In case (2), temperature \( T \) can either have a phase delay with respect to \( q \) or be ahead of it depending on the period of external forcing \( P = \frac{2\pi}{\omega_R} \) at RF periods from one year to hundreds of years. \( q \) has a lag relative to \( T \) (\( \Delta T_q > 0 \)), while at periods from hundreds to tens of thousands of years, \( T \) has a lag relative to \( q \) (\( \Delta T_q < 0 \)). When the periods approach hundreds of thousands of years, a lag of \( q \) relative to \( T \) appears again; however, it is not significant compared to the period of \( P \) (Fig. 2).

The delay of \( q \) relative to \( T \) on different time scales was also found in [10]. However, the delay of \( T \) relative to \( q \) over secular and millennium periods was not found in [10]. The latter may be related to a less detailed account for the inertia characteristics of the terrestrial carbon stock in the suggested model compared with [10].

It is noteworthy that the detailed analysis of the ice cores also demonstrates opposite phase shifts for the modes with periods of approximately 4^0 years or less [2, 3] along with the general lag of variations in \( q \) relative to the variations in \( T \) for the glaciation
cycles, whose periods are estimated at approximately 100,000 years.

Numerical experiments were performed for case (2), in which we used fixed values of evasion factor $\zeta$ and CO$_2$ solubility in seawater $\chi$ to calculate the CO$_2$ flux into the ocean (see (5)). If we neglect the temperature dependence of solubility and constants of other chemical reactions, which participate in the non-organic carbon cycle in the ocean, the manifestation of mutual delay does not change qualitatively. This agrees with the conclusions about the dominating role of the anthropogenic emissions in the accumulation of CO$_2$ in the atmosphere in the present time period related to the release of this gas by the ocean during warming.

In case (3), the phase shift was determined only from the correlation coefficient between the time series of the first differences because the correlation coefficient between time series of $q$ and $T$ has no pronounced maximum (over the entire numerical experiment $q$ and $T$ increase with superimposed small fluctuations caused by periodical radiative forcing). The manifestation of mutual delay is similar to case (2) discussed above.

This result can be generalized over a wide class of periodical forcing (in particular with the extension to Fourier series). It is worth noting that the value of the external forcing period $P$ on the system, during which time lag $\Delta T_q$ changes its sign, generally speaking depends on its form as a time function because each harmonic (mode) has its own phase shift.

Thus, we can conclude that during climate warming the time lag between the variations in the global temperature and the concentration of CO$_2$ in the atmosphere due to emissions determined from the maximum of the correlation between their first differences (as it was done, for example, in [4]) is completely determined by radiative forcing. In particular, this means that the effect of time lag of CO$_2$ concentration relative to the variations in the global temperature described in [4] is not a sufficient cause to state that the former is not the cause of the latter and the results obtained in [4] do not contradict the conclusions about the anthropogenic nature of modern global warming [1].

Our results can be explained if we consider the initial linearized model:

\[
\frac{dq}{dt} = (\beta_{oc} + \beta_{land}) + \alpha D + A_R M + \gamma T + E(t),
\]

\[
\frac{dD}{dt} = -\beta_{oc} q - \alpha D,
\]

\[
\frac{dM}{dt} = -\beta_{land} q - A_R M - \gamma T,
\]

\[
\frac{dT}{dt} = R q + \nu T + R_f(t).
\]

Our results can be explained if we consider the initial linearized model:

Here,

\[
\beta_{oc} = -F_0, \quad \beta_{land} = -A_p g'_s(q_0),
\]

\[
\alpha = \zeta(T_0, q_0) \frac{q_0}{D_0}, \quad M = M_b + M_s,
\]

\[
\gamma = -(2\theta_p - \theta_b - \theta_s), \quad R = \frac{R_x}{C q_0}, \quad \nu = -\frac{\lambda}{C}.
\]

It is possible to get analytical expressions for $\Delta T_q$ that have different signs at different values of the equation parameters, in particular, for different periods of external forcing $P$.

When $R_f(t) = R_0 \sin(\omega_R t)$ and $E(t) \equiv 0$ (case 2), the solution of system (10)–(13) is found in the following form:

\[
q = q_0 \sin(\omega_R t + \varphi_q),
\]

\[
\Delta T_q = \frac{\Delta T}{P} = \frac{\Delta T}{P}.
\]
and when \( R_f(t) = R_0 \sin(\omega_R t) \) and \( E(t) = E_0 \exp \left( \frac{t}{\tau_E} \right) \) (case 3), the solution is found in the form

\[
q = q_e \exp \left( \frac{t}{\tau_E} \right) + q_s \sin(\omega_R t + \varphi_q),
\]

\[
T = T_e \exp \left( \frac{t}{\tau_E} \right) + T_s \sin(\omega_R t + \varphi_T).
\]

Here, \( \varphi_q \) is the phase shift of \( q \) relative to \( R_f(t) \), \( \varphi_T \) is the phase shift of \( T \) relative to \( R_f(t) \), and \( \Delta q = \varphi_T - \varphi_q \). The changes in the solutions during exponential emissions are related to aperiodical terms \( q_e \exp \frac{t}{\tau_E} \) and \( T_e \exp \frac{t}{\tau_E} \), which do not influence significantly the value of the time lag determined from the maximum of the correlation coefficient. In the limit case \( \tau_E = \infty \), which corresponds to constant emissions; the expressions for the first differences between \( q \) and \( T \) coincide. This explains a similar manifestation of the time lag between variations in \( q \) and \( T \) in cases (2) and (3).

One can show (if we neglect the dependence of \( F_{oc} \) on \( D \)) that when

\[
\omega < \omega_0 = \left( \beta_{oc} \nu \right)^{1/2}
\]

\( \varphi_q > 0 \), i.e., variations in \( q \) are ahead of the radiative forcing that causes them (for the realistic values of \( \beta_{oc} \) and \( \nu \), the value of \( 2\pi / \omega_0 \) is of the order of a few hundred years). This qualitatively agrees with the results of the numerical experiments (Fig. 2).

It is worth noting that the mutual delay between the analyzed variables is determined by cause-and-effect links between them and by the time scales related to these variables [11]. In the model considered above, along with the time scales of external forcing \( \left( \frac{2\pi}{\omega_R}, \frac{2\pi}{\omega_E}, \text{and} \tau_E \right) \), one can distinguish time scales, one of which is related to the thermal inertia of the ES and can be estimated at \( \tau_T = \nu^{-1} = \frac{C}{\lambda} \sim 30 \text{ years} \), and the second can be related to the response of the \( \text{CO}_2 \) concentration in the atmosphere to the external forcing, which is equal to \( \tau_q = (\beta_{oc} + \beta_{land})^{-1} \) (approximately 30 years for the oceanic reservoir and 10 years for the land basin) and also the time scale related to the interaction between the climate and carbon cycle, which is equal to \( \tau_c = 2\pi [(\beta_{oc} + \beta_{land}) \nu]^{-1/2} \), i.e., of the order of magnitude of a few hundred years. The scale of the thermal inertia \( \tau_T \) characterizes the asymptotic value \( \Delta q \) during anthropogenic emissions of \( \text{CO}_2 \) into the atmosphere with a long (relative to \( \tau_T \) period

\[
P = \frac{2\pi}{\omega_E} \exp \frac{t}{\tau_E}.
\]

The asymptotic value of \( \Delta q \) in the case of external radiative forcing can be estimated by the total time lag related to \( \tau_f \) and \( \tau_q \).

It is worth noting that in the model considered here when the RF periods are of the order of 1000 years and longer, the sign of \( \Delta q \) also changes depending on the sign of parameter \( \gamma = \left( 2\theta_p - \theta_b - \theta_s \right) \), which in turn can change during small variations in parameters \( \theta_p \), \( \theta_b \), and \( \theta_s \). This means that interpretation of the sign of the time lag between the variations in \( q \) and \( T \) obtained from the data of observations depends on the sign of \( \gamma \). The parameter \( \gamma \) is a parameter of the feedback between the climate and the carbon cycle (\( \gamma > 0 \) means that climate warming facilitates the increase in the concentration of \( \text{CO}_2 \) in the atmosphere). In the major part of the modern models, this feedback is positive [12], and in this work we considered only the case when \( \gamma > 0 \).

Thus, we demonstrated that the time lag of the variations in the carbon dioxide concentration in the atmosphere \( q \) relative to the variations in the global surface temperature \( T \) obtained from the data of the paleo-reconstructions can also be reproduced using the generally accepted climatic models and does not contradict the conclusions about the key role of the anthropogenic greenhouse effect in the modern climate changes. We also obtained that variations in \( q \) can either delay relative to the variations in \( T \) or be ahead of them depending on the type of external forcing on the ES (radiative forcing or external emissions of \( \text{CO}_2 \) into the atmosphere), the period of this forcing, and the characteristics of the feedback between the climate and carbon cycle.

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